Decomposing stock return autocorrelation into spurious and genuine components☆

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Abstract

We present methods for decomposing stock return autocorrelation into spurious components— the nonsynchronous trading effect (NT) and bid-ask bounce (BAB)—and genuine components—partial price adjustment (PPA) and time-varying risk premia (TVRP). Our methods are applicable to any return period (e.g., daily, weekly, monthly, annual returns, etc.), to both individual stock and portfolio return autocorrelations, and to any time horizon (over which the autocorrelations are calculated). The tests are direct in the sense that they are not dependent on any particular market microstructure model of PPA. The tests are constructed using the following four key ideas: theoretically signing or bounding the various components in different situations; computing returns over disjoint subperiods (return periods) separated by a trade to eliminate NT and greatly reduce or eliminate BAB; dividing the data period into disjoint subperiods (time horizons) to obtain independent measures of autocorrelation; and computing the portion of the autocorrelation that can be unambiguously attributed to PPA. We apply our methods to daily individual and portfolio return autocorrelations on the New York Stock Exchange (NYSE) over a data period of ten years, divided into five two-year time-horizon subperiods; our analysis indicates that TVRP is a negligible source of autocorrelation in this setting. We find that PPA is an important source, and in some cases the main source, of the daily return autocorrelation of individual stocks and of portfolios, especially among small- and medium-size firms. We find that a very substantial fraction of the total autocorrelation arises from PPA.

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1. Introduction

There is an extensive literature concerning the autocorrelation of stock returns. The autocorrelation patterns have been attributed to four main sources: spurious autocorrelation arising from market microstructure biases, including the nonsynchronous trading effect (NT) (in which correlations are calculated using stale prices) and bid-ask bounce (BAB), and genuine autocorrelation arising from partial price adjustment (PPA) (i.e., trade takes place at prices that do not fully reflect the information possessed by traders) and time-varying risk premia (TVRP).[1] There has been considerable controversy over the proportion of the autocorrelation that should be attributed to each of these four components, in part because previous methods for decomposing the autocorrelation into its constituent parts have been indirect. There have been direct tests of the speed of price adjustment and PPA, but not of the role of PPA in stock return autocorrelation.[2] Indeed, there has been controversy over whether the autocorrelation even exists: Chordia et al. (2005) write: "Daily returns for stocks listed on the New York Stock Exchange (NYSE) are not serially correlated;" if that were true, this paper would be considerably shorter.

In this paper, we propose several methods to decompose individual stock and portfolio return autocorrelation into these four components. Each of these methods uses one or a combination of the following four key ideas:

Sign and/or bound the sources of autocorrelation theoretically:

- NT is negative for individual stock returns, and is generally positive for portfolio returns.
- BAB is negative for both individual stock and portfolio returns, and is generally considered to be very small for portfolio returns.[3]
- PPA can be either positive or negative for both individual stock and portfolio returns.[4]
- TVRP is positive for both individual stock and portfolio returns. In Appendix B, we calculate an upper bound on the size of TVRP, depending on the return period (daily, weekly, monthly,

quarterly, annual returns) and the time horizon over which the autocorrelations are calculated. We find that TVRP is too small to affect the tests in this paper, which focus on daily return autocorrelation over time horizons of two years; consequently, in the discussion of our tests, we shall assume there are only three sources, NT, BAB, and PPA, for daily return autocorrelation.[5] If we find statistically significant positive autocorrelation in individual stock returns, and it is too large to be explained by TVRP, then it must arise from PPA. If we find statistically significant negative autocorrelation in portfolio returns, it must be either PPA or BAB, and it is much more plausible that the autocorrelation arises from negative PPA than from BAB.

Eliminate NT by computing returns over disjoint return subperiods, separated by a trade:

NT arises when autocorrelations are computed using stale prices. If we compute stock returns in a way that stale prices are never used, NT will be eliminated.

- For individual daily stock returns, NT arises when a stock is not traded for several days, so the return is taken to be zero, then the stock trades and incorporates several days worth of trend all in one day. Conventional daily return on a given day is defined as the price of the last trade on that day, minus the price of the last previous trade, divided by the price of the last previous trade. We define the *intraday return* on a given day to be the price of the last trade on that day, minus the price of the first trade on that day, divided by the price of the first trade on that day. Any catch-up to trend will be reflected in the first trade of the day, and consequently will not be incorporated into the intraday return. On days in which fewer than two trades occur, we treat the data as missing rather than zero. This eliminates NT, and greatly reduces BAB.
- Portfolio return autocorrelation is not the average of the individual return autocorrelations of stocks in the portfolio. Instead, it is dominated by cross-autocorrelations of the returns of the individual stocks in the portfolio. NT arises when information arrives during a given day. For stocks that trade after the information arrives, the information is reflected in the return that day; for stocks that do not trade after the information arrives, the information is reflected in the return

the next day, so we see positive cross-autocorrelation in the individual stocks in the portfolio, generating positive portfolio return autocorrelation. We define the intraday portfolio return on a given day to be the average of the intraday returns for that day of the individual stocks in the portfolio. We then define the intraday return autocorrelation of the portfolio to be the autocorrelation of the intraday portfolio returns. This eliminates NT, and eliminates or greatly reduces whatever small amount of BAB there may be in portfolio returns.

- Exchange-traded funds (ETFs) are mutual funds that trade in real-time during the market day, rather than only once per day at net asset value. Many ETFs, especially those based on broad market indices, trade essentially continuously. SPDRs are an ETF based on the Standard and Poor's 500 index (S&P500). Since the return autocorrelation of a portfolio is essentially the average of the return *cross*-autocorrelations of the pairs of stocks in the portfolio, we compute the cross-autocorrelation of tomorrow's conventional return on a stock with today's SPDR return up to the SPDR trade immediately preceding today's last trade of the stock, then average over firms in the portfolio. This eliminates NT, and eliminates or greatly reduces whatever small amount of BAB there may be in portfolio returns.

Divide the data period into disjoint time-horizon subperiods to achieve independent measures of autocorrelation:

Binomial models cannot be used to aggregate tests of autocorrelation of individual stocks in a single data period, because returns are correlated across stocks. However, in all conventional models of stock pricing, return autocorrelations are independent across disjoint time periods. Consequently, if we divide our data period into disjoint time-horizon subperiods, then tests within the various subperiods *can* be aggregated using binomial models or other methods that require independence:

- Within each time-horizon subperiod, compute the individual return autocorrelations of each stock in a group, then average across stocks in the group and test whether the average return is statistically significant. Count the number of subperiods in which the average is statistically significant, and test this using a binomial model.

- Within each time-horizon subperiod, the number of stocks with statistically significant individual return autocorrelation is a nonnegative random variable with known mean. The probability that this number exceeds any given level can be bounded by the equivalent for means of the Chebychev inequality for variances. Compute the number in each subperiod, then compute the order statistics of the subperiod counts. The probability that the first (smallest) order statistic exceeds a certain level, and the probability that the second (second smallest) order statistic exceeds a given level, can be determined from the binomial distribution, based on the Chebychev-like inequality described earlier.
- Within each time-horizon subperiod, compute the portfolio return autocorrelation, and test whether it is statistically significant. Count the number of subperiods in which the autocorrelation is statistically significant, and test this using a binomial model.

Compute the proportion of the autocorrelation clearly attributable to PPA:

Using the three key ideas above, we can identify a portion of the autocorrelation that can only be attributed to PPA, a portion that cannot be PPA, and a portion that may or may not be PPA. Take the portion that can only be attributed to PPA, and compute the residual (total autocorrelation less the portion that can only be attributed to PPA; equivalently, the sum of the portion that cannot be PPA and the portion that may or may not be PPA). Compute the absolute value of the portion that can only be attributed to PPA, divided by the sum of the absolute values of the portion that can only be attributed to PPA and the absolute value of the residual. This gives a lower bound on the portion of the autocorrelation arising from PPA.

Data set and findings:

We examine ten years (1993-2002) of transaction data of stocks listed on the NYSE. We find strong evidence that PPA plays a substantial role in daily individual stock and portfolio return autocorrelation in all of our tests involving small and medium firms, and in some of our tests involving large firms. PPA must be a significant source, and in some cases, the main source, of autocorrelation in stock returns.

As noted above, the specific tests we use are based on combinations of the four key ideas we have just explained. The following two subsections outline our tests and findings, for individual stock returns and portfolio returns.

1.1. Individual stock returns

NT and BAB both produce negative autocorrelation in daily individual stock returns. Previous studies have tested the average daily return autocorrelation over all stocks in a given market, and have generally found that average to be statistically insignificant. Chan (1993) reports average daily return autocorrelation in deciles of NYSE and American Stock Exchange (AMEX) stocks, finding that average autocorrelations are negative and significant for small firms; insignificant for medium firms; and positive and significant for large firms, while noting that the grand average among firms of all size is not significant. Although he does not note this, Chan's data provide strong evidence for PPA for large firms in the 1980s, even though the grand average autocorrelation is not significant. We shall explain this more fully in Section 4.1.1.

Our analysis focuses on the five two-year time-horizon subperiods of our ten-year data period 1993-2002. The *average* (within each size group) of the individual *conventional* stock return autocorrelations in one time-horizon subperiod is independent of the average in the other time-horizon subperiods. We test the hypothesis that this average is nonpositive in each subperiod, and find that it is rejected at the 1% level for small, medium and large firms. We conclude that PPA is an important source of the autocorrelation in stock returns among firms of all sizes.

The conventional return autocorrelation of each individual stock in one time-horizon subperiod is independent of that of each individual stock in the other time-horizon subperiods. We test the hypothesis that the conventional daily return autocorrelation of *each* individual stock is nonpositive in each subperiod. We find that this hypothesis is rejected at the 1% level among small and (depending on which test of autocorrelation we use) at the 1% or 5% level for medium firms. For these firms, there must be something other than NT and BAB at work, and PPA is the only plausible candidate. We conclude that PPA is present in these firms, and that its effect is larger than the combined effect of NT and BAB.

In addition to studying the autocorrelation of conventional daily stock returns, we study the autocorrelation of the *intraday* return. The use of intraday returns eliminates NT and eliminates or greatly reduces BAB in stock return autocorrelation. The use of intraday returns also eliminates some of the PPA in stock return autocorrelation, but provides us with a direct measure of the remaining portion of the autocorrelation.

Breaking our data period into five two-year time-horizon subperiods, we find that the *average* intraday return autocorrelation is positive and overwhelmingly significant among small and medium firms; this is strong evidence of the existence of PPA as a source of autocorrelation, and that it is positive on average. For large firms, we find strong evidence that the average intraday return autocorrelation is negative in at least some subperiods; this most plausibly reflects negative PPA (overshooting) arising from momentum traders, with the overall sign of PPA varying between periods, depending on the relative numbers of informed and momentum traders.

We reject (at the 1% level for small firms, and (depending on which correlation test we use) at the 1% or 5% level for medium firms) the hypothesis that the intraday return autocorrelation of *each* firm is nonpositive; this is strong evidence that PPA is an important source of stock return autocorrelation among small and medium firms. For small, medium, and large firms, the hypothesis that the autocorrelation of intraday return of every stock is nonnegative is not rejected; indeed, the fraction of stocks with negative intraday return autocorrelation is systematically below the expected value of 2.5%. We conclude that PPA is systematically positive for small and medium stocks. As with the average intraday return autocorrelation, the number of positive and negative return autocorrelations among large firms varies from period to period.

Our methods allow us to estimate a lower bound on the portion of the identifiable absolute autocovariance arising from PPA: 56.2% for small firms, 60.7% for medium firms, and 52.6% for large firms.

1.2. Portfolio returns

BAB produces slightly negative autocorrelation in daily portfolio returns, while NT produces positive autocorrelation; PPA can be either positive or negative. Thus, the finding that daily portfolio returns exhibit positive autocorrelation is not sufficient to establish that PPA is a source of the autocorrelation. A number of papers have carried out indirect tests that tend to support the role of PPA in autocorrelation, but the results have been controversial because of the indirect nature of the tests.

In this paper, we conduct two direct tests of the roles of NT and PPA in explaining the positive autocorrelation in daily portfolio returns.

In the first test, we define the intraday return of a portfolio as the equally-weighted average of the intraday returns of the individual stocks in the portfolio.[6] Since the intraday return of a portfolio on a given day depends only on trades that occur that day, NT is eliminated. We find that the autocorrelation of conventional portfolio returns is positive and strongly significant for small, medium and large firms. The autocorrelation of intraday portfolio returns is positive and highly significant for small and medium firms, providing strong evidence of PPA. The autocorrelation of intraday portfolio returns is negative and not significant for large firms.

For the second test, note that the return autocorrelation of a portfolio of 100 stocks is the average of 10,000 autocorrelations: 9,900 cross-autocorrelations and 100 own-autocorrelations. Thus, the portfolio return autocorrelation is essentially the average of the return cross-autocorrelations of the pairs of stocks. In the second test, we compute the cross-correlation of daily returns on SPDRs up to the time of the last trade of a given stock, and that stock's next-day conventional return. In this setting, NT is eliminated, but BAB remains and is negative. Our main null hypothesis is that the cross-correlation is less than or equal to zero for every stock in the portfolio. This hypothesis is strongly rejected for all three portfolios and all three (Pearson, Andrews' modification of Pearson, and Kendall tau) correlation tests.

Our method allows us to estimate a lower bound on the proportion of portfolio return autocorrelation arising from PPA: 54.6% for small firms, 59.5% for medium firms, and 36.8% for large firms.

The remainder of this paper is organized as follows. Section 2 details our methodology and null hypotheses. Section 3 describes the sampling of firms and provides descriptive statistics of our data. Section 4 presents and interprets the empirical results. Section 5 provides a summary of our results and some suggestions for further research.

2. Methodology

As noted by Lo and MacKinlay (1990), NT arises from measurement error in calculating stock returns. If an individual stock does not trade on a given day, its daily return is reported as zero; if it does not trade for several days, it is in effect accumulating several days of unreported gain or loss, which is captured in the data on the first subsequent day on which trade occurs. Think of the "true" price of the stock being driven by a positive trend plus a daily volatility term (with mean zero), with the reported price being updated only on those days on which trade occurs. On days on which no trade occurs, the reported return will be zero, which is below trend; on days on which trade occurs after one or more days without trade, the reported return represents several days' worth of trend; this results in spurious negative autocorrelation. Even if a stock does trade on a given day, the reported "daily closing price" is the price at which the last transaction occurred, which might be several hours before the market closed. Thus, a single piece of information that affects the underlying value of stocks i and j may be incorporated into the reported price of i today because i trades after the information is revealed, but not incorporated into the reported price of j until tomorrow because j has no further trades today, resulting in a positive cross-autocorrelation between the prices of i and j. Hence, NT causes spurious negative individual autocorrelation and positive individual cross-autocorrelation, resulting in positive autocorrelation of portfolios. The first key idea in this paper is to theoretically sign and/or bound the various sources of autocorrelation, so that we may draw inferences about the source from the sign of the observed autocorrelation.

Since Fisher (1966) and Scholes and Williams (1977) first pointed out NT, the extent to which it can explain autocorrelation has been extensively studied, but remains very controversial.

The second key idea in this paper is to study stock returns over *disjoint* time intervals where a *trade occurs between the intervals*. More formally, we study the correlation of stock returns over intervals $[s,t]$ and $[u,v]$ with $s < t \leq u < v$ such that the stock trades at least once on the interval $[t,u]$. We apply this idea to derive tests in a number of different situations. Because these correlation calculations do not make use of stale prices, NT is, *by definition*, eliminated; if the correlation turns out to be nonzero, there must be a source, other than NT, for the correlation. This conclusion does not depend on any particular story of how the use of stale prices results in spurious correlation.

In addition to eliminating NT, our method of calculating correlations also eliminates or greatly reduces BAB in many of the situations in which we apply it.

We say that a stock exhibits PPA if there are trades at which the trade price does not fully reflect the information available at the time of the trade. Let r_{sti} denote the return on stock i (i=1,…,I) over the time interval $[s,t]$; in other words, $r_{sti}=\frac{S_i(t)}{S_i(s)}-1$, where $S_i(t)$ is the price of stock i at the last trade occurring at or before time t. Let F_t denote the σ-algebra representing the information available at time t. Since the stock price at each trade is observable, $S_i(t)$ must be F_t-measurable. The absence of PPA in stock j implies the following:[7]

given times $s < t \leq u < v$ such that stock j trades at some time $w \in [t,u]$, r_{uvj} is uncorrelated with every random variable which is F_w-measurable, and hence uncorrelated with r_{stj}.

Thus, we can test for the presence or absence of PPA by examining return correlations over time intervals $[s,t]$ and $[u,v]$ satisfying the condition just given.

Two of our tests focus on what we call intraday returns; in these tests, NT is eliminated, and BAB is eliminated or greatly reduced. The intraday return of a stock on a given day is defined as the price of the last trade of the day, less the price at the first trade of the day, divided by the price at the first trade of the day. Thus the intraday return of stock i on day d is $r_{s_i t_i i}$, where s_i and t_i are the times of the first and last trades of the stock on day d.[8] We compute the correlation $\rho(r_{s_i t_i i}, r_{u_i v_i i})$, where u_i and v_i are the times of the first and last trades on day d+1. Note that $s_i < t_i < u_i < v_i$, so NT is eliminated. BAB arises in conventional daily return autocorrelation because the correlation considered is $\rho(r_{q_i t_i i}, r_{t_i v_i i})$, where q_i is the time of the last trade prior to day d. Note that the end time in calculating $r_{q_i t_i i}$ is the same as the starting time in calculating $r_{t_i v_i i}$, resulting in negative autocorrelation, as explained in Roll (1984); Roll's model assumes that at each trade, the toss of a fair coin determines whether the trade occurs at the bid or ask price. In the calculation of the intraday autocorrelation, the end time t_i of the first interval is different from the starting time u_i of the second interval. Moreover, the trades at t_i and u_i are different trades, so the coin tosses for these trades are independent; if we apply Roll's model to this situation, the autocorrelation resulting from BAB is zero. If we extend Roll's model to multiple stocks, and assume the coin tosses are independent across stocks, the autocorrelation and cross-autocorrelation of intraday stock returns are zero. Relaxing the independence assumption results in slightly negative autocorrelation and cross-autocorrelation of intraday returns.[9]

Any single correlation can be tested by a variety of standard methods; the three methods we use are outlined below.

The third key idea in this paper is to divide the data period into disjoint time-horizon subperiods, and use the fact that autocorrelations within disjoint time-horizon subperiods are independent, so we can derive tests using the binomial distribution. This idea is applied in two different settings. In the first setting, we compute a single autocorrelation in each subperiod; in one case, we compute the average of individual stock return autocorrelations over each subperiod, while in another case, we compute the portfolio return autocorrelation over each subperiod. We count the number of time-horizon subperiods in which the single autocorrelation value is statistically significant, and use the binomial distribution to test for significance overall. For example, the probability of a one-sided rejection in any given subperiod using a symmetric 5% rejection criterion is 2.5%, so the probability of obtaining two rejections in five subperiods is $(5!/(2!3!))(.025)^2=.00625$, leading to overall rejection at the 1% level.

In the second setting, we apply the binomial distribution to counts of stocks with statistically significant return autocorrelations in each of the time-horizon subperiods. Depending on the test, we reject the hypothesis for an individual firm in a given time-horizon subperiod using either a symmetric 5% rejection criterion or a one-sided 2.5% rejection criterion. If the correlation tests were independent across firms, the number of rejections would have the binomial distribution. If the collection $\{ r_{s_i t_i i} : i = 1,\ldots,I \}$ were a family of independent random variables, then X, the number of firms for which the zero-correlation hypothesis is rejected at the 5% (2.5%) level, would be binomially distributed, as $B(I,0.05)$ ($B(I,0.025)$), which has mean $0.05I$ ($0.025I$) and standard deviation $\sqrt{(.05)(.95)I}$ ($\sqrt{(.025)(.975)I}$). Since returns are not independent across stocks, X will not be binomial. The standard deviation of X is not readily ascertainable, and is likely higher than that of the binomial. However, the failure of independence does not change the mean of X, so X is a nonnegative, integer-valued, random variable with mean $0.05I$ ($0.025I$).

In all of these tests, there are I=100 firms, so X has mean μ=5 or μ=2.5. Since X is nonnegative, $P(X \geq \alpha\mu) \leq 1/\alpha$ for every $\alpha \geq 1$. Suppose that we compute X in each of n disjoint time-horizon subperiods. This provides us with n independent observations of X; let $X_1,\ldots,X_n$ be the order statistics, i.e., X_1 is the smallest observation, X_2 the second smallest, and so forth. Then using the binomial distribution, for every $\alpha \geq 1$, $P(X_1 \geq \alpha\mu) \leq 1/\alpha^n$ and $P(X_2 \geq \alpha\mu) \leq 1/\alpha^n + n(1-1/\alpha)/\alpha^{n-1} = (n\alpha - (n-1))/\alpha^n$. Given particular realizations $x_1 \geq \mu$ and $x_2 \geq \mu$ of X_1 and X_2, we obtain *p*-values of $p_1 = 1/(x_1/\mu)^n$ for x_1 and $p_2 = (n(x_2/\mu) - (n-1))/(x_2/\mu)^n$ for x_2, respectively. The test for the k^{th} order statistic X_k involves the combinatorial coefficient $n!/((k-1)!(n-k+1)!)$ as well as the factor $(\mu/x_k)^{n-k+1}$, both of which grow rapidly with k. Thus, the power of the test for X_k declines rapidly with k, suggesting the test be based on X_1 alone. However, the test for X_1 can be strongly affected by a single outlier. In particular, if any single realization of X is less than μ, then p_1=1 and the null hypothesis will not be rejected. For these reasons, we adopt a combined test using both X_1 and X_2, and not using the higher order statistics. Compute the statistic $p_3 = 2\, min\{p_1,p_2\}$. Note that for any γ, $P(p_3 \leq \gamma) = P(2\, min\{p_1,p_2\} \leq \gamma) = P(p_1 \leq \gamma/2$ or $p_2 \leq \gamma/2) \leq P(p_1 \leq \gamma/2) + P(p_2 \leq \gamma/2) = \gamma/2 + \gamma/2 = \gamma$. Thus, the *p*-value in the combined test is $p_3 = 2\, min\{p_1,p_2\}$. Note that p_3 depends on μ and n.

We analyzed ten years' worth of data. There is a trade-off between the number of time-horizon subperiods and the lengths of the time-horizon subperiods. Because stock returns are very noisy, for a given return period, it is much easier to detect autocorrelation in longer time-horizon subperiods than in shorter time-horizon subperiods. On the other hand, the statistical power of the order statistic tests increases when the number of independent observations (the number of time-horizon subperiods) increases. Some of our results are statistically stronger when the analysis is done with five two-year periods, while others are statistically stronger with ten one-year periods; on the whole, the results are qualitatively similar.

2.1. Individual stock returns

Studies of autocorrelation in individual stock returns have focused on the average autocorrelation of groups of firms, finding it to be statistically insignificant and usually positive; see Säfvenblad (2000) for a survey. This finding is consistent with two possibilities: either the autocorrelation of each individual stock is essentially zero; or some stocks exhibit positive autocorrelation and others exhibit negative autocorrelation, with the two largely canceling out when averaged over stocks. None of the previous studies analyzed the autocorrelation of individual stocks one by one. Doing so is essential for testing whether the autocorrelation arises from NT or PPA, or both.

In this paper, we average the autocorrelation over groups of firms, segregated by firm size, and also consider the autocorrelations of individual firms. We calculate the autocorrelation in two different ways: the conventional daily return autocorrelation, and the intraday return autocorrelation.

2.1.1. Conventional daily return autocorrelation

For each firm, we calculate the daily return on each day in the conventional way: the closing price on day *d*, minus the closing price on the last day prior to day *d* on which trade occurs, divided by the closing price on the last day prior to day *d* on which trade occurs. When we compute individual stock returns in the conventional way, NT and BAB are both present, and both generate negative autocorrelation. Hence, in the absence of PPA, every firm must exhibit daily return autocorrelation less than or equal to zero, and consequently the average daily return autocorrelation in a group of stocks must be less than or equal to zero. Null Hypothesis I (IA, IB) is that the *average* daily return autocorrelation is zero (nonpositive, nonnegative) in each firm group in each of our five two-year time-horizon subperiods; we test these hypotheses by comparing the average daily return autocorrelation for each subperiod to the associated standard error. Rejection of Null Hypothesis IA implies that there is PPA, and that it is positive in at least some subperiods. Rejection of Null Hypothesis IB implies that, on average, the sum of NT, BAB, and PPA is negative in at least some subperiods.

Because the average autocorrelations are independent across time-horizon subperiods, the number of subperiods on which a hypothesis is rejected has the binomial distribution. In each subperiod, we use a symmetric 5% criterion for rejection, so the one-sided rejection criteria are at the 2.5% level. As noted above, we reject Null Hypothesis IA (IB) at the 1% level if the average daily return autocorrelation is statistically positive (negative) in two or more of the five subperiods.

We also compute the average autocorrelation over the whole ten-year period as a weighted average of the period averages, weighting each period by the inverse square of the associated standard error. We report the averages and standard errors for the individual time-horizon subperiods, as well as the weighted average and standard error over the whole ten-year period.

Null Hypothesis II is that *every* firm's daily returns exhibit zero autocorrelation. For each firm, we test whether daily returns exhibit zero autocorrelation, in each of n=5 disjoint two-year time-horizon subperiods. In each subperiod, we test whether the sample autocorrelation of each stock lies in a symmetric 95% confidence interval that places 2.5% probability on each of the two tails, so μ=5. Applying this test to 100 firms in each of the 5 subperiods, we reject Null Hypothesis II if $p_3 = p_3(\mu=5, n=5) < 0.05$. Null Hypothesis IIA is that each firm exhibits nonpositive autocorrelation. For each firm, we test whether daily returns exhibit nonpositive autocorrelation, in each of the five disjoint two-year subperiods. In each subperiod, we test whether the sample autocorrelation lies above the 97.5 %-ile, so μ=2.5.[10] Applying this test to 100 firms in each of the 5 disjoint subperiods, we reject Null Hypothesis IIA if $p_3 = p_3(\mu=2.5, n=5) < 0.05$. Rejection of Null Hypothesis IIA implies that in at least some firms, the PPA is positive and, indeed, is larger than the sum of the NT and BAB effects.

Null Hypothesis IIB is that each firm exhibits nonnegative autocorrelation. For each firm and each time-horizon subperiod, we test whether the sample autocorrelation lies below the 2.5 %-ile; we reject Null Hypothesis IIB if $p_3 = p_3(\mu=2.5, n=5) < 0.05$. Rejection of Null Hypothesis IIB implies that in some firms, the sum of the NT, BAB, and PPA effects is less than or equal to zero. This is consistent with there being a negative PPA effect for this group of stocks, or a positive PPA effect which is outweighed by the NT and BAB effects.

2.1.2. Intraday return autocorrelation

As above, we define the intraday return on day d as the price at the final trade on day d, minus the price at the first trade on day d, divided by the price at the first trade on day d. If a given stock does not trade, or has only one trade, on a given day, we drop the observation of that stock for that day from our data set.[11] If we compare intraday returns on day d and day d+1, there is *no* NT effect: the intraday returns are computed over disjoint time intervals, with each interval beginning and ending with a trade, so stale prices never enter the calculation. Moreover, because the first trade on day d+1 is a different trade from the last trade on day d, BAB is eliminated or greatly reduced. If NT and BAB are the sole sources of stock return autocorrelation, the theoretical autocorrelation of intraday returns on each stock must be less than equal to zero, and close to zero. This implies that the average autocorrelation of intraday returns on each group of stocks must be less than or equal to zero and close to zero. Null Hypothesis III (IIIA, IIIB) is that the *average* autocorrelation of intraday returns is zero (nonpositive, nonnegative) in each of the five two-year time-horizon subperiods in each group of stocks. In each subperiod, we test whether the subperiod autocorrelation is significantly nonzero (positive, negative) at the 5% level by comparing the autocorrelation to its standard error. As in the case of Null Hypotheses IA (IB), we reject Null Hypothesis IIIA (IIIB) at the 1% level if the autocorrelation is statistically positive (negative) in two or more of the five subperiods. Rejection of Null Hypothesis IIIA implies that there is PPA, and it is positive on average in at least some subperiods. Rejection of Null Hypothesis IIIB implies that there is PPA, and that it is negative on average in at least some subperiods. As in the case of conventional returns, we also compute the average autocorrelations over the entire ten-year period.

Our Null Hypothesis IV is that the autocorrelation of intraday returns on *each* stock is zero, Null Hypothesis IVA is that the autocorrelation of intraday returns on each stock is nonpositive, and Null Hypothesis IVB is that the autocorrelation of intraday returns on each stock is nonnegative. The criteria for rejection of Null Hypotheses IV, IVA, and IVB are identical to those of Null Hypotheses II, IIA, and IIB, except that we use intraday returns rather than conventional daily returns. In particular, we divide our data period into five two-year time-horizon subperiods, and test using $p_3(\mu=5,n=5)$ for Null Hypothesis IV, and $p_3(\mu=2.5,n=5)$ for Null Hypotheses IVA and IVB. Because intraday returns exhibit neither NT nor BAB, rejection of Null Hypothesis IV implies that PPA is a source of individual stock autocorrelation. Rejection of Null Hypothesis IVA implies that the autocorrelation arising from PPA is positive, arising from slow incorporation of information into prices more than from overshooting due to positive-feedback strategies.

2.1.3. Analysis of autocovariance

The fourth key idea in this paper is to use the decomposition of autocorrelation into its various components to estimate the fraction of the autocorrelation arising from PPA. In this section, we describe a method to obtain a lower bound on the portion of the individual stock autocovariance attributable to PPA. Conventional daily returns are calculated from the closing trade one day to the closing trade of the next day on which trade occurs; the union of these intervals, from one closing trade to the next, covers our data period 24 hours per day, 7 days per week. However, the intraday returns of the stocks are calculated over a portion of the data period, namely the union of the intervals of time beginning with the first trade of a stock on a day and the last trade of the same stock on that day. A portion of the period when the markets are open, and the entire period when the markets are closed, are omitted. In all conventional models of stock pricing, the standard deviation of intraday return should be lower than the standard deviation of conventional daily return. For example, if the stock price is any Itô Process, the price changes over the excluded intervals are uncorrelated with the price changes over the included intervals. Since the variance of a sum of uncorrelated random variables is the sum of the variances, the exclusion of the intervals must decrease the variance. Notice that this argument applies to the theoretical variance—the variance of the theoretical distribution of returns. The observed variance of returns for a given stock is the variance of a sample out of that theoretical distribution of returns, so the standard deviation of intraday return might be larger than the standard deviation of conventional daily return for a few stocks. In our sample, we find that only 21 of the 1,500 stocks (300 stocks per time-horizon subperiod times five subperiods) exhibits sample standard deviation of intraday returns greater than the sample standard deviation of conventional daily return.

For each stock, we can compute the conventional daily (intraday) return autocovariance by taking the product of the conventional daily (intraday) return autocorrelation times the conventional daily (intraday) return variance. Note that these autocovariances can be either positive or negative, so it is not appropriate to compute their ratio. However, we know that PPA is the only source of the intraday return autocovariance. If C_i and I_i denote the conventional daily and intraday return autocovariances of stock i, let $W_i = C_i - I_i$ denote the residual. W_i, C_i, and I_i may each be either positive or negative. Thus, we consider $\frac{|I_i|}{|I_i| + |W_i|}$ as the fraction of the identifiable absolute autocovariance arising from intraday returns. This ratio is a lower bound on the portion of the identifiable return autocorrelation attributable to PPA. It understates the proportion of the autocorrelation attributable to PPA for two reasons. First, PPA can induce both negative and positive effects; these cancel, and we see only the net effect in this calculation. Second, PPA occurring between the last trade of a stock on a given day and the first trade on the next day is also omitted from this calculation.

2.2. Portfolio returns

Atchison et al. (1987) and Lo and MacKinlay (1990) find that NT explains only a small part of the portfolio autocorrelation (16% for daily autocorrelation in Atchison et al., 0.07, a small part of the total autocorrelation, for weekly autocorrelation in Lo and MacKinlay). However, Boudoukh et al. (1994) find that the weekly autocorrelation attributed to NT in a portfolio of small stocks is as high as 0.20 (56% of the total autocorrelation) when the standard assumptions by Lo and MacKinlay are loosened by considering heterogeneous nontrading probabilities and heterogeneous betas.[12] This leads them to conclude that "institutional factors are the most likely source of the autocorrelation patterns."

The use of intraday data has led to renewed interest in this issue. For example, Ahn et al. (2002) comment that Kadlec and Patterson (1999), using intraday data and simulation, find that "nontrading can explain 85%, 52%, and 36% of daily autocorrelations on portfolios of small, random, and large stocks, respectively. In other words, nontrading is important *but not the whole story* [italics added]." Ahn et al. assert that the positive autocorrelation of portfolio returns "can most easily be associated with market microstructure-based explanations, as partial [price] adjustment models do not seem to capture these characteristics of the data."

There has also been support for PPA. Chan (1993) provides a model in which there is a separate market-maker for each stock; each market-maker observes a signal of the value of his/her stock, and sets the price at the correct conditional expectation, given the signal, so that individual stock returns show no autocorrelation; and stock returns exhibit positive cross-autocorrelation, because the signals are correlated across stocks. He tests some predictions of this model, finding support for positive cross-autocorrelation, and for his prediction that the cross-autocorrelation is higher following large price movements. Chordia and Swaminathan (2000) compare portfolios of large, actively traded stocks, to portfolios of smaller, thinly traded stocks, arguing that NT should be more significant in the latter than in the former. The data they report on the autocorrelations of these portfolios "suggest that nontrading issues cannot be the sole explanation for the autocorrelations […] and other evidence [concerning the rate at which prices of stocks adjust to information] to be presented." Llorente et al. (2002) relate the volume to the autocorrelation, arguing that the relative importance of hedging and speculative trading determines the direction of the relationship, with positive autocorrelation arising if speculative trading (in which informed agents slowly exercise their informational advantage) predominates.[13]

While many papers have studied whether NT can fully explain positive portfolio autocorrelation, all of the tests have been indirect. In this paper, we propose and carry out two direct tests that eliminate NT. In both tests, we compute the correlation of returns of securities over disjoint time intervals separated by a trade, so that stale prices never enter the correlation calculation. If NT and BAB are the sole explanations of portfolio return autocorrelation, the autocorrelation computed by our methods must be less than or equal to zero.

2.2.1. First method, intraday returns

In the first method, we compute the intraday returns of each individual stock as defined in Section 2.1.2. As noted there, intraday returns on different days do not exhibit NT, and BAB should be eliminated or greatly reduced. We consider three portfolios, each containing 100 stocks, representing small, medium, and large market capitalization.

We define the intraday return of a portfolio on a given day as the equally-weighted average of the intraday returns for that day on all stocks in the portfolio, omitting those stocks which have fewer than two trades on that day. Note that the autocorrelation of the intraday return of the portfolio is just the average of the correlations of the intraday returns of the individual pairs (including the diagonal pairs) of stocks in the portfolio. Since 99% of these pairs are off-diagonal, the portfolio return autocorrelation is dominated by the cross-autocorrelations between pairs of stocks. In particular, the portfolio return autocorrelation is *not* the average of the individual return own autocorrelations of the stocks in the portfolio.

If NT and BAB are the sole sources of stock return autocorrelation, the autocorrelation of the intraday return of the portfolio must be less than or equal to zero, and close to zero. Thus, our Null Hypothesis V is that the autocorrelation of the intraday return of the portfolio is zero. Rejection of Null Hypothesis V implies that there is a nonzero PPA effect; the sign of the PPA effect is determined by the sign of the autocorrelation. As in the case of average individual stock autocorrelations, we test the portfolio return autocorrelation separately in each of our five two-year time-horizon subperiods, and compute the portfolio return autocorrelation for the whole ten-year period as a weighted average of the subperiod results; we report both the subperiod results and the weighted average, along with the associated standard errors.

The computation of the autocorrelation of the intraday return of the portfolio allows us to obtain a lower bound on the portion of the conventional daily return autocorrelation attributable to PPA. As in Section 2.1.3, all conventional models of stock pricing predict that the variance of intraday portfolio returns should be lower than the variance of conventional daily portfolio returns; we find that this is the case in each of the three portfolios and each of the five two-year time-horizon subperiods in our data set. We calculate the autocovariance of conventional daily (intraday) portfolio returns by multiplying the conventional daily (intraday) autocorrelation of portfolio returns by the conventional daily (intraday) variance of portfolio returns. The residual is defined as the difference of the conventional and intraday autocovariances. The autocovariance of intraday portfolio returns can only come from PPA, so the ratio of the intraday autocovariance to the sum of the absolute values of the intraday and residual autocovariances gives a lower bound on the proportion of the autocorrelation that is attributable to PPA.

2.2.2. Second method, ETFs

In the second method, we take our portfolio to be an ETF. ETFs are continuously-traded securities which represent ownership of the stocks in a particular mutual fund or index. Because a mutual fund is valued once a day, and an index is calculated at any given instant by averaging the most recent price of each stock in the index, and some of those prices are stale, the mutual funds and indices are themselves subject to NT. For example, the quoted value of the S&P 500 index exhibits stale pricing because it is an average of the most recent trade price of the stocks in the index (see Kimelman (2003)). ETFs are traded continuously and very actively, the value is updated continuously, rather than with lags arising from intervals between trades of the underlying stocks. At any instant, each stock price is somewhat stale because it has not been adjusted since the last trade, so the index exhibits staleness; however, each trade of the ETF represents an actual trade, which by definition is not stale at the time it occurs. In particular, each trade of the ETF occurs at a price different from the current value of the index; in the absence of PPA, the ETF price should reflect all the information in the market, in particular the "correct" price of the stocks in the index, even if many of those stocks have not traded for some time.

For this paper, the ETF we choose is SPDRs, an ETF based on the S&P 500 index; each SPDR share represents a claim to one-tenth of the value of the S&P 500 index. In our sample period, SPDRs exhibit weakly negative daily autocorrelation. The daily return of each individual stock on day d is computed in the conventional way: the price at the final trade on day d, minus the price at the last trade prior to day d, divided by the price at the last trade prior to day *d*. We compute the correlation between the conventional return of stock *i* on day *d*+1 (in other words, the return over the return period from the final trade of the stock on day *d* to the final trade of the stock on day *d*+1) with the return of the SPDRs over the return period from the time of the last trade of the SPDRs on day *d*-1 through the time of the last trade of the *stock* on day *d*. If a stock does not trade on day *d* or the stock does not trade on day *d*+1, we omit the data from our calculation.[14] Note that each time we compute a correlation, it is the correlation of a stock return over a given return period with the return of a traded security, SPDRs, over a disjoint return period, with both the SPDRs and the stock trading in the interval separating the two return periods. Thus, the calculation of the correlation does not use stale prices, and hence there is no NT effect. There may be an effect due to BAB, but if so, it should be negative. Thus, in the absence of PPA, the correlation between the return of the individual stock and the return of the SPDRs must be less than or equal to zero. Our Null Hypothesis VI (VIA, VIB) is that the correlation of each of the individual stock returns and the return of the SPDRs is zero (nonpositive, nonnegative). As in Null Hypotheses II (IIA, IIB) and IV (IVA, IVB), we divide our data period into five two-year time-horizon subperiods, and test using $p_3(\mu=5,n=5)$ for Null Hypothesis VI, and $p_3(\mu=2.5,n=5)$ for Null Hypotheses VIA and VIB. Rejection of Null Hypothesis VIA implies that the PPA exists and is positive.

In our test, the only detectable source of PPA is the slow incorporation into the price of individual firms of the very public, non-firm-specific, information contained in the price of SPDRs. The remainder, which presumably constitutes the vast majority of the total PPA present in the market, is not captured by these tests.

Ex ante, this seems to us an unlikely place to search for PPA. PPA is usually discussed in the market microstructure literature, and is understood to mean the slow incorporation of private, firm-specific, information into the prices of individual securities. The current price of SPDRs is public, not private. Indeed, because of its link to the closely-watched S&P 500 index, it is one of a handful of the most visible market statistics. Other studies of the incorporation of public information into securities prices, and their relationship to this paper, are discussed in footnote 2. The two studies most closely related to our SPDR tests are Hasbrouck (1996, 2003). Hasbrouck (2003) showed that index futures lead SPDRs in incorporating new information. Hasbrouck (1996) showed that order flows for large individual stocks generated by index futures-based program trading and index arbitrage have price impacts beyond the index futures themselves, but that these are very quickly incorporated; as indicated in his Figure 1, the impact is substantially achieved within two minutes, and completely achieved within three minutes. Indeed, the two-minute effect is quite close to the permanent effect, and the three-minute effect overshoots. Since index futures lead SPDRs, individual stocks should lag SPDRs by a shorter interval.[15] These relatively short lags seem to us unlikely to result in daily return autocorrelation, for two reasons. First, stock prices are very volatile and it seems likely that the autocorrelation resulting from a three-minute lag would be swamped by the variability over the other 387 minutes of the trading day. Second, the R^2 of overall market returns on the returns of individual stocks is quite low. Since the correlation of SPDRs with individual stocks induced by the lag will be the product of the information revealed in the lag times the R^2, the effect should be very small indeed. To test whether the lags documented by Hasbrouck (1996, 2003) can explain our findings, we rerun our tests for large firms, calculating the SPDR return over a return period ending three minutes before the last trade of the stock.

Because the definitions underlying Null Hypotheses VI, VIA, and VIB are somewhat complex, we here present a more formal statement of the model.

Assuming closing time is 4:00 p.m., we define the following notation:

$$S_{(d,h)i} \text{ = Price of stock } i \text{ at hour } h \text{ on date } d, \tag{1}$$

$$\bar{S}_{(d,h)} \text{ = Price of SPDRs at hour } h \text{ on date } d, \tag{2}$$

$$h(d,i) \text{ = Hour of last trade of stock } i \text{ on date } d, \tag{3}$$

$$S_{di} = S_{(d,h(d,i))i} \text{ (the closing price)}, \tag{4}$$

$$\bar{S}_d = \bar{S}_{d,4pm}, \tag{5}$$

$$r_{di} = \frac{S_{di} - S_{(d-1)i}}{S_{(d-1)i}}, \tag{6}$$

$$\bar{r}_d = \frac{\bar{S}_d - \bar{S}_{d-1}}{\bar{S}_{d-1}}, \quad (7)$$

where "hour" means actual time of transaction; thus, it indicates transaction data down to the minute and second.

We decompose the daily return of the SPDRs, $\frac{\bar{S}_d - \bar{S}_{d-1}}{\bar{S}_{d-1}}$, into two components, $\frac{\bar{S}_d - \bar{S}_{d,h(d,i))}}{\bar{S}_{d-1}}$ and $\frac{\bar{S}_{d,h(d,i)} - \bar{S}_{d-1}}{\bar{S}_{d-1}}$. No stale prices are used in the calculation of $Corr\left(r_{(d+1)i}, \frac{\bar{S}_{d,h(d,i)} - \bar{S}_{d-1}}{\bar{S}_{d-1}}\right)$, the correlation between the return of stock i tomorrow and today's return of SPDRs up to the time of the stock i's today's last transaction. These two returns are computed on disjoint return periods separated by trades, as we can see in Fig. 1, so there is no NT effect; for details, see Appendix A. Consequently, in the absence of PPA and BAB, this covariance must be zero. Since BAB induces negative autocorrelation, in the absence of PPA, the correlation must be less than or equal to zero.

$$Corr\left(r_{(d+1)i}, \frac{\bar{S}_{d,h(d,i)} - \bar{S}_{d-1}}{\bar{S}_{d-1}}\right) \le 0. \quad (8)$$

<Insert Fig. 1>

2.3. Testing hypothesis: testing for covariance

Testing each of our Null Hypotheses requires testing whether a correlation or a set of correlations is zero, positive, or negative. We use three test methods: the Pearson correlation test, the modified Pearson correlation test using Andrews'(1991) heteroskedasticity and autoregression consistent (HAC) covariance estimator, and the Kendall tau test.

- ***Pearson correlation test* (*a parametric test*):** This method tests whether the correlation between two variables is zero, positive, or negative using the Pearson product-moment correlation coefficient. Letting r_p be the Pearson sample correlation coefficient, the t-test statistics are

$$t = r_p \sqrt{\frac{n-2}{1-r_p^2}} \ \sim \ t(n-2). \tag{9}$$

This test assumes that the variables have a bivariate normal distribution.

- ***Modified Pearson correlation test*****:** We use Andrews' modification of the Pearson test, taking into account the possibility that the error terms exhibit heteroskedasticity or autocorrelation. We use Andrews' (1991) HAC covariance estimator to estimate the correlation coefficient and to test whether it is zero, positive, or negative. The test is based on the fact that the *t*-test statistic of the correlation coefficient of the two variables is numerically equal to the *t*-statistic on the regression coefficient of one variable with respect to the other. The HAC covariance is obtained using Andrews' quadratic spectral (QS) kernel with automatic bandwidth selection method.
- ***Kendall tau test* (*a nonparametric test*):** Stochastic volatility biases the standard errors in the Pearson correlation test. The Kendall tau test is a nonparametric test that makes no assumptions on the joint distribution of the variables, and is completely immune to the effects of stochastic volatility. Kendall's sample rank correlation coefficient is defined by

$$\hat{\tau} = \frac{2K}{n(n-1)}, \tag{10}$$

where $K = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} Q\left((X_i, Y_i), (X_j, Y_j)\right)$ and $Q((a,b),(c,d)) = \begin{cases} 1, & if\,(c-a)(d-b) > 0 \\ -1, & if\,(c-a)(d-b) < 0 \end{cases}$. Then the Kendall tau test statistic is given by

$$T = 3\hat{\tau}\frac{\sqrt{n(n-1)}}{\sqrt{2(2n+5)}} \ \sim \ N(0,1), \tag{11}$$

which is asymptotically normal; the normal provides an excellent approximation provided that $n > 10$.[16]

3. Data

Our data period covers the ten calendar years 1993 through 2002. We divide this into five two-year time-horizon subperiods: 1993-94, 1995-96, 1997-98, 1999-2000, and 2001-02. Within each two-year subperiod, we obtain a sample of 100 small, 100 medium, and 100 large firms.

Because our data period contains the market bubble that burst in 2000, one might be concerned about "regression to the max" (Ross (1987), Ross et al. (1995)). These papers point out that if researchers deliberately focus on periods immediately before and after a big local maximum or minimum in securities prices, a sample selection bias arises in the computation of return autocorrelation. For example, the paths of Geometric Brownian Motion exhibit substantial local maxes and mins. If one takes a long history of Geometric Brownian Motion, then computes the autocorrelation only in the time-horizon subperiods in which a large local max or min occurred, one is likely to find positive autocorrelation that appears statistically significant in those subperiods, as a result of the bias. However, if one selects the data period without regard to local maxes and mins, there is no bias.

We selected our ten-year data period in a kosher way.[17] Of course, the inadvertent inclusion of the bubble might have meant that our results would not extend to other periods. Could the inclusion of the bubble have affected our results? No. All of our autocorrelations are computed separately in each of the two-year time-horizon subperiods 1993-94, 1995-96, 1997-98, 1999-2000, 2001-02. Only one of those subperiods, 1999-2000, contains a local max or min that stands out in comparison to other time periods. As a robustness check, we recomputed our *p*-values using only the other four subperiods. The *p*-values rise somewhat because less data is being used, but virtually all of our results retain statistical significance.

The samples are obtained using the following criteria:

- Since our analysis requires firms' market capitalization, we select the sample from the set of common stocks included in both the Trade and Quote (TAQ) database master file and the Center for Research in Security Price (CRSP) tapes for the subperiod; the trade data comes exclusively from TAQ, while we use CRSP data to establish market capitalization. We exclude closed end investment companies or trusts from our set of common stocks.

- We remove firms in which the total number of shares outstanding changes by more than ten percent during the two-year time-horizon subperiod because changes in the number of shares outstanding could significantly affect the frequency of trade and in particular the time interval between the last trade of the day and the market close.[18]
- We remove firms for which no transaction occurs for 30 or more consecutive trading days within the two-year time-horizon subperiod.
- We remove firms whose shares traded for less than $5 at any point in the two-year time-horizon subperiod. We did this in order to eliminate financially distressed firms. We verified manually that most of them did eventually become penny stocks.
- We eliminate any stock whose shares traded for more than $1,000 at any point in the two-year time-horizon subperiod; only one stock (Berkshire Hathaway) was removed by this process.
- We form three different groups of firms stratified by market capitalization. For each stock, we calculate the market capitalization by multiplying the number of shares outstanding by the daily closing price (or the average of the bid-ask quotes) on the last day of trading preceding the time-horizon subperiod. The large-firm sample consists of the 100 largest firms not eliminated by the criteria above; 97 (average for the five two-year time-horizon subperiods) of the original firms were eliminated. The medium-firm sample consists of the 100 firms with market capitalizations closest to the median that were not eliminated by the criteria above; 120.2 of the original firms were eliminated. For the small-firm sample, following Bessembinder (1997), we eliminated the 50 smallest, in order to avoid including an unduly large number of financially distressed firms in the sample; we then took the smallest 100 firms not eliminated by the criteria above; 268 were eliminated at this stage.

For each of these 100 NYSE-listed firms in a given group and time-horizon subperiod, we obtain transaction data from the TAQ database; we exclude trades that occurred on other exchanges. We manually cleaned the data to remove clearly erroneous prices.[19] We also use transaction data of the

SPDRs over the same sample data period. Since our goal is to understand the sources of daily autocorrelation in stock returns, and daily returns are calculated from the closing market prices, we exclude trade that occurs in the after-hours markets from our data set. Thus, for each individual stock, we compute the closing price in the conventional way: the last transaction price reported *before* 4:00 p.m.

Table 1 reports descriptive statistics of our 300 NYSE-listed sample firms stratified into each of three groups: small, medium, and large firms. The variables are the number of firms, the firms' market capitalization (in millions of dollars; max, mean, and min), average daily trading volume (in shares), average time interval between the last transaction and the market close (in seconds), and average number of days on which trade occurs. The figures reported are the averages over the five two-year time-horizon subperiods, except for max and min where we report the max of the subperiod maxes and the min of the subperiod mins. The numbers of Table 1 reflect the diverse sample of securities used in this study. The firms' market values range from 23.9 million dollars to 475.0 billion dollars. The Table shows that as the firm size increases, both average trading volume and average trading days increase. Table 1 also reports the mean daily returns of each portfolio; 0.0604%, 0.0395%, and 0.0348% for small-, medium-, and large-firm portfolio respectively, reflecting a strong small-firm effect. For the first-order autocorrelation of conventional daily portfolio returns for each portfolio, see Table 6.

The closing trade for an individual stock is the last trade occurring before 4:00 p.m. In order to ensure that the closing trade of the SPDRs occurs after the closing trade of each stock, we take the closing trade of the SPDRs to be the first trade occurring after 4:00 p.m., except on the 26 days on which the market closed early, where we take the closing trade of the SPDRs to be the last trade occurring before 4:00 p.m. For individual stocks as well as SPDRs, the closing price is defined to be the transaction price of the closing trade. On average, conditional on there being at least one transaction in a given stock, the last transaction of the small, medium, and large firms occurs 47.3 minutes, 11.4 minutes, and 1.2 minutes, respectively, before the closing trade of the SPDRs.[20]

For SPDRs, the first-order daily return autocorrelation is slightly negative (-0.0199) but statistically insignificant. Since SPDRs are traded continuously, we do not expect to find positive daily return autocorrelation arising from NT, as we see in portfolios; however, SPDRs are subject to BAB.

<Insert Table 1>

4. Empirical results and their implications

4.1. Individual stock returns

4.1.1. Conventional daily return autocorrelation

Table 2 reports results on the conventional daily return autocorrelation of individual firms, stratified into the three groups. Null Hypothesis IA, that the average conventional daily return autocorrelation is nonpositive in each of the five time-horizon subperiods, is rejected at the 1% level among small, medium, and large firms. As noted in Section 2.1.1, we reject Null Hypothesis IA at the 1% if the autocorrelation is statistically positive at the 5% level (2.5% one-sided level) in two or more of the five subperiods. The fact that medium and large firms exhibit insignificant or negative autocorrelation in some subperiods suggests that the relative importance of PPA, NT, and BAB varies somewhat from period to period. Since both the NT and BAB effects result in negative autocorrelation, this finding provides compelling evidence that PPA is the main source of daily return autocorrelation among small firms, and an important source of daily return autocorrelation among medium and large firms.

In the model of Chan (1993), the daily return autocorrelation of each individual firm is zero, which implies our Null Hypothesis I. In Chan's Table I, he reports that the average autocorrelation for all NYSE and AMEX firms was positive and highly significant in the period 1980-84, negative and highly significant in the period 1985-89, and not significant over the entire period 1980-89. He also found that average daily return autocorrelation was negative and highly significant for small firms, not significant for medium firms, and positive and highly significant for large firms in the period 1980-89. Although he does not note this, Chan's results are not consistent with zero daily return autocorrelation in each firm; in particular, the results reported in his Table I imply rejection of all three of our Null Hypotheses I, IA, and IB. While the average return autocorrelation of all stocks is insignificant over his entire data period, the systematic effects of firm size and year on the autocorrelation provide clear evidence that there is more going on in the market than in Chan's model.

<Insert Table 2>

Table 3 reports the results for Null Hypotheses II, IIA, and IIB. Null Hypothesis II, that each firm's daily return autocorrelation is zero, is rejected at the 1% level, for all three correlation tests, for both small and medium firms. Null Hypothesis IIA, that each firm's daily return autocorrelation is nonpositive, is rejected at the 1% level, for all three correlation test, for small firms; it is rejected at the 5% level, for two of the three correlation tests, and the 1% level for the remaining test, for medium firms. The strong rejection of Null Hypothesis IIA shows that the NT and BAB effects, which predict negative daily return autocorrelation for individual firms, cannot be the only source, or even the main source, of autocorrelation in small and medium firms.

Null Hypothesis IIB, that each firm's daily return autocorrelation is nonnegative, is rejected at the 5% level, for all three correlation tests, in medium firms. It is not rejected among large firms, and is rejected by only one of the three correlation tests among small firms. The rejection among medium firms could indicate that the PPA effect is negative for some stocks, due to overshooting. Alternatively, it could result from negative NT and BAB effects that more than offset the positive PPA.

In summary, all of our tests provide strong evidence that PPA plays an important role in individual stock return autocorrelation among small and medium firms.

<Insert Table 3>

4.1.2. Intraday return autocorrelation

Table 4 reports results on average intraday return autocorrelation of individual firms, stratified into the three groups. As noted above, intraday returns are calculated so as to eliminate NT and eliminate or greatly reduce BAB. The only plausible source of the remaining autocorrelation is PPA. Averages are reported for each of the five two-year time-horizon subperiods, along with an average for the whole ten-year period obtained as a weighted average of the subperiod averages.

Null Hypothesis IIIA is overwhelmingly rejected among small and medium firms.[21] This indicates that the average (over firms in the small and medium groups) PPA is significant and positive in at least some time-horizon subperiods; in fact, it is positive in all five subperiods for small firms and four out of five subperiods for large firms. Null Hypothesis IIIB is rejected at the 1% level among large firms, indicating that the average (over large firms) PPA is significant and negative in at least some of the subperiods. This presumably reflects negative autocorrelation arising from overshooting, due to momentum traders, with the balance between slow price adjustment and overshooting shifting back and forth across periods in response to the relative numbers of informed and momentum traders.

<Insert Table 4>

Table 5 reports our results on individual intraday stock returns. Null Hypothesis IV, that each firm's intraday return autocorrelation is zero, is rejected at the 1% level, for all three correlation tests, for small and medium firms; it is not rejected among large firms.

As noted above, Null Hypothesis IVA, that each firm's intraday return autocorrelation is nonpositive, is the most important hypothesis in this part of our study. Null Hypothesis IVA is rejected at the 1% level for all three correlation tests for small firms. It is rejected at the 1% level for two of the three correlation tests for medium firms, barely missing the 1% level in the remaining test. This shows that PPA is a significant source of daily return autocorrelation in small and medium stocks.

Null Hypothesis IVB, that each firm's intraday return autocorrelation is nonnegative, is not rejected, for any of the correlation tests, in any of the three size groups. This indicates that the PPA effect is systematically positive among small and medium firms: the negative autocorrelation resulting from positive-feedback strategies and overshooting is systematically smaller than the positive autocorrelation resulting from the slow incorporation of information into prices.

The numbers of positive and negative rejections among large firms vary significantly among time-horizon subperiods, preventing us from rejecting either Null Hypothesis IVA or IVB. This is consistent with our conjecture that variation in the number of traders using momentum strategies explains the autocorrelation pattern among individual large stock returns.

Finally, we use the methodology described in section 2.1.3 to provide a lower bound of the identifiable absolute autocovariance of individual stocks arising from PPA. Over the five two-year time-horizon subperiods, our estimates range from 48.0% to 61.8% (average 56.2%) for small stocks, 50.5% to 64.6% (average 60.7%) for medium stocks, and 48.8% to 56.7% (average 52.6%) for large stocks. In all three firm-size groups, more than half of the autocovariance of individual stocks comes from PPA.

<Insert Table 5>

4.1.3. Summary–Individual stock return autocorrelation

PPA must be a important source of the autocorrelation of daily returns of individual stocks, among small and medium firms. PPA is systematically positive among small and medium, indicating that the positive autocorrelation arising from slow incorporation of information into prices outweighs the negative autocorrelation arising from positive-feedback strategies and consequent overshooting. The most likely explanation of our findings among large firms is that PPA is also an important source of autocorrelation of daily returns, but the sign of PPA varies among time-horizon subperiods, possibly as a result of variation in the popularity of momentum strategies among traders of large stocks.

4.2. Portfolio returns

4.2.1. First method, intraday returns

Tables 6 and 7 report our results concerning conventional and intraday portfolio returns. Results are presented for each of the five two-year time-horizon subperiods, along with a value for the whole ten-year period computed as a weighted average of the subperiod returns. Because conventional portfolio returns do not provide a test of PPA, we did not formalize null hypotheses on conventional portfolio returns. Table 6 presents the results for conventional portfolio returns. The conventional daily return autocorrelations of small-, medium-, and large-firm portfolios are positive and significant at the 1% level for all three correlation tests. As the firm size becomes larger, the first-order autocorrelation of portfolio return becomes smaller. This result is consistent with those of the previous studies (e.g., Chordia and Swaminathan (2000, Table I on page 917)). Table 7 presents the results for intraday portfolio returns, and our tests of Null Hypothesis V. The intraday portfolio return autocorrelation is positive and significant at the 1% level, for all three correlation tests, for small and medium firms. This provides strong evidence that PPA is an important source of portfolio return autocorrelation, and that it is on balance positive, in small and medium firms. Among large firms, the conventional portfolio return autocorrelation is positive and significant, although smaller than among medium and small firms. By contrast, the intraday portfolio return is not significant and slightly negative. Thus, our intraday portfolio returns do not provide evidence of PPA in portfolios of large stocks; however, as we shall see, we do find such evidence in our test involving SPDRs.

Table 8 shows the autocovariances of conventional (intraday) returns in each of the three portfolios, obtained by multiplying the conventional (intraday) return autocorrelations by the conventional (intraday) variances. The ratio of the intraday autcovariance to the sum of the absolute values of the intraday and residual autocovariances ranges from 44.8% to 65.5% over the five time-horizon subperiods, with an average of 54.6%, for small firms; from 5.0% to 90.8%, with an average of 59.5%, for medium firms, and from 18.4% to 64.0%, with an average of 36.8%, for large firms. As noted above, these figures represent lower bounds of the portion of the autocorrelation attributable to PPA.

<Insert Table 6>

<Insert Table 7>

<Insert Table 8>

4.2.2. Second method, ETFs

Table 9 shows the results of our tests of Null Hypotheses VI (VIA, VIB): that the correlation of individual stock and SPDRs returns is zero (nonpositive, nonnegative). As explained above, these correlations are calculated in a way that eliminates NT but not necessarily BAB. Consequently, if NT and BAB are the only sources of stock return autocorrelation, the correlation must be less than or equal to zero. Thus, Null Hypothesis VIA is the most important of this group of hypotheses. For all three correlation tests, it is rejected at the 1% level for small and medium firms, and at the 5% level for large firms. The rejection of Null Hypothesis VIA provides strong evidence of PPA among small, medium, and large firms.

It is important to note, also, that Null Hypothesis VIB, that the correlation of individual stock and SPDRs returns is nonnegative, is not rejected for any of the correlation tests or size groups of firms.

When we look at the five two-year time-horizon subperiods, a somewhat more nuanced story emerges. The ratio of positive to negative rejections varies substantially among subperiods, particularly among large stocks. In some subperiods, the number of negative rejections is very low, and this results in the failure to reject Null Hypothesis VIB. However, there are subperiods where the negative rejections outnumber the positive rejections, indicating again that the correlation pattern between the SPDRs and firms varies somewhat over time, perhaps with the prevalence of program trading to arbitrage discrepancies between the prices of the ETF and its underlying stocks.

<Insert Table 9>

As noted above, Hasbrouck (1996, 2003) has documented that prices of ETFs lead prices of the constituent stocks, but the lags are small, not more than three minutes. To test whether these short lags could be explaining our results for large stocks, we reran the tests of Null Hypotheses VI (VIA, VIB), computing the SPDR return up through the last SPDR trade at least three minutes before the closing trade of the stock. The results are reported in Table 10. The counts of stocks with statistically significant correlation change very little, and the *p*-values either stay the same or increase very slightly. Null

Hypothesis VIA is still rejected at the 5% level for the modified Pearson and Kendall tau tests, and just misses (p=.0625) for the Pearson test. Null Hypothesis VI is rejected for all three correlation tests. We conclude that the lags documented by Hasbrouck cannot explain our findings.

Our finding of PPA for large firms in the SPDRs tests contrasts with the failure to find such evidence for large firms in the portfolio intraday return tests. In the SPDR tests, the time interval between the final trade of a stock and the last previous SPDR trade is just over three minutes. By contrast, in the portfolio intraday return tests, the time interval between the first trade on day d+1 and the last trade on day d includes an entire overnight period, as well as some time when the markets are open, and it appears that large stock prices do adjust to information overnight.

<Insert Table 10>

5. Concluding remarks

We present methods for decomposing stock and portfolio return autocorrelation into NT, BAB, PPA, and TVRP. These methods make use of four key ideas: theoretically signing and/or bounding these components; computing returns over disjoint return subperiods separated by a trade to eliminate NT; subdividing our data period into time-horizon subperiods to obtain independent measures of autocorrelation; and isolating a portion of the autocorrelation that can only come from PPA to obtain a lower bound on the portion of the autocorrelation attributable to PPA. We find compelling evidence that PPA is an important source, and in some cases the main source, of stock return autocorrelation. PPA is an important source of autocorrelation in all of our tests involving small and medium firms, and in some tests involving large firms. Our tests cover both individual stock return autocorrelation and portfolio return autocorrelation. In contrast to earlier tests of the role of PPA in stock return autocorrelation, our tests are direct.

We use two methods to eliminate NT. The first method computes correlations of intraday returns;

this method can be applied to eliminate NT with other types of securities, and on other exchanges. The second method, used in computing the correlation of individual stock returns and SPDRs, computes the return of the SPDRs separately in the periods before and after the final trade of the stock; this method can be used to eliminate NT using any security which, like the SPDRs, is traded nearly continuously.

By dividing our data period into disjoint time-horizon subperiods, we obtain independent tests of the sources of autocorrelation in the different time-horizon subperiods. Aggregating the tests across time-horizon subperiods allows us to increase the statistical power of our tests, and to work around the problem that returns are correlated across stocks. Our methods for aggregating the results of the time-horizon subperiod tests can be applied to other types of securities and other exchanges.

Further research is needed on the following questions:

- to what extent do these findings extend to other markets involving different institutional structures?
- among large firms, we find strong evidence of PPA among portfolios in our test using SPDRs, but not our tests involving intraday returns. The use of intraday returns allows us to measure only a portion of the PPA; that portion is large enough to generate statistical significance among small and medium firms, but evidently not among large firms. The test involving SPDRs captures a different portion of the PPA. Is there some other way to capture more of the PPA in a single test?
- our tests seem to indicate that PPA among large firms is positive in certain periods and negative in other periods. We argue that this most plausibly reflects variations in the relative numbers of informed and momentum traders. Is there a way to test this?
- among small and medium firms, all our tests provide strong evidence of the importance of PPA in individual return autocorrelation. Among large firms, we find similar evidence in the test aggregating average autocorrelation tests in the time-horizon subperiods, but not from our analysis of individual firm return autocorrelations. The number of positive autocorrelations of

individual large-firm intraday returns in the various time-horizon subperiods is usually substantially above the expected value, suggesting that PPA plays a role in return autocorrelation in this setting, but our test involving the first two order statistics is too weak to detect it, at least using five two-year or ten one-year time-horizon subperiods. Would a longer overall data period, and/or a different statistical test, allow one to demonstrate the role of PPA in stock return autocorrelation in this setting?

Appendix A: Derivation of equation (8)

As we can see in Fig. 1, the daily return of SPDRs at day d, $\frac{\bar{S}_d - \bar{S}_{d-1}}{\bar{S}_{d-1}}$, consists of two components, $\frac{\bar{S}_{d,h(d,i)} - \bar{S}_{d-1}}{\bar{S}_{d-1}}$ (the return of the SPDRs from 4:00 p.m. yesterday (day d-1), to time $h(d, i)$ today (day d)) and $\frac{\bar{S}_d - \bar{S}_{d,h(d,i))}}{\bar{S}_{d-1}}$ (the return from time $h(d, i)$ today to 4:00 p.m. today). Here $h(d, i)$ is the time of the individual stock i's last transaction on day d. We have the following identity:

$$\frac{\bar{S}_d - \bar{S}_{d-1}}{\bar{S}_{d-1}} = \frac{\bar{S}_d - \bar{S}_{d,h(d,i))}}{\bar{S}_{d-1}} + \frac{\bar{S}_{d,h(d,i)} - \bar{S}_{d-1}}{\bar{S}_{d-1}}. \tag{12}$$

For individual stock i, the last transaction occurs at time $h(d, i)$ of day d and $h(d+1, i)$ of day d+1. The usual story for correlation arising from NT goes as follows: Suppose that information affecting the value of the stock i becomes known between $h(d, i)$ and 4:00 p.m. of day d (interval B in Fig. 1). This information will not be reflected in stock i's closing price on day d, but will be reflected in price on day d+1, and thus in the return, $r(d+1, i)$, *on day* d+1. However, the SPDRs trade very frequently, and will usually trade at many times between $h(d, i)$ and 4:00 p.m. of day d. Consequently, the information will be reflected in the SPDRs' price and return *on day d*. This induces a spurious positive correlation between the SPDRs' return on day d and the stock return on day d+1.

Our analysis, however, is not dependent on the particular mechanism by which NT induces spurious correlation. The contribution of the NT effect to the correlation between the SPDRs return and the stock return comes solely from the interval B in Fig. 1, where there is an overlap between the time intervals on which the day d return of the SPDRs and the day d+1 return of stock i are computed. Said slightly differently, the return of the stock on day d+1 is computed using the price of the stock at the time of its last trade on day d, and that price is stale on the interval B in Fig. 1, but is *not* stale at the time $h(d,i)$.

$r_{(d+1),i}$ is the return over the intervals B and C. $\frac{\bar{S}_d - \bar{S}_{d-1}}{\bar{S}_{d-1}}$ is the return over the intervals A and B. The correlation comes only from the overlap, interval B. If we eliminate interval B from our return calculation for the SPDRs, the return becomes $\frac{\bar{S}_{d,h(d,i)} - \bar{S}_{d-1}}{\bar{S}_{d-1}}$. In the correlation $Corr\left(r_{(d+1)i}, \frac{\bar{S}_{d,h(d,i)} - \bar{S}_{d-1}}{\bar{S}_{d-1}} \right)$, no stale prices are used; if the correlation is not zero, it must be coming from something other than NT. Since BAB induces negative correlation, in the absence of PPA, the correlation must be less than or equal to zero, so Equation (8) holds.

Appendix B: Time-varying risk premia

In this Appendix, we analyze the effect of TVRP on stock return autocorrelation. This allows us to estimate the magnitude of the effect on our autocorrelation estimates, and thereby validate our estimates of PPA.

Under the assumption that stock prices follow one of the standard processes in finance (such as a Geometric Itô or Geometric Lévy Process), rejection of the hypothesis that stock return autocorrelation is zero is equivalent to rejection of the hypothesis that the expected rate of return is constant. In other words, if we impose the assumption that the return in each period is composed of an expected rate of return plus a volatility term, where the volatility term is uncorrelated with the returns in disjoint periods, then returns are uncorrelated if and only if the expected rate of return is constant. As noted by Campbell et al. (1997, page 66), the "R^2 of a regression of returns on a constant and its first lag is the square of the slope coefficient, which is simply the first-order autocorrelation." As a consequence, if the first-order autocorrelation coefficient of return is α, the proportion of the variation in return that "is predictable using the preceding day's ... return" is α^2. Thus, time-varying expected rates of return and return autocorrelation are simply different faces of a single phenomenon.

However, time-varying expected rates of return and TVRP are distinct phenomena. To illustrate, suppose that stock prices follow Itô Processes of the form $\frac{dS}{S} = \mu\, dt + \sigma\, dW$; a similar analysis holds if prices follow other standard processes. The absence of arbitrage is equivalent to the existence of a vector process λ of prices of risk such that $\mu - r = \sigma\lambda'$; here, μ is the vector process of expected rates of return and r is the risk-free rate. The expected rate of return μ will vary as a result of changes in r, σ, and λ, and the resulting variation in μ cannot be exploited by arbitrage; this is the variation attributable to TVRP. Other variation in μ constitutes time-varying expected rates of return, not TVRP; it can be exploited by arbitrage.

Standard autocorrelation tests are designed to test for time-varying expected rates of return, but cannot distinguish TVRP from other forms of time-varying expected rates of return. In particular, if a stock has a run of positive returns, autocorrelation tests will conclude that the stock had a high expected rate of return over that period, but cannot distinguish whether or not this is the result of a high risk premium. If it is, then it cannot be exploited by arbitrage by informed traders.

If the high expected rate of return is not the result of a high risk premium, then it *can* be exploited by arbitrage. If no one knew the expected rate of return was high, there would be nothing pushing the stock higher, and it would stay relatively stable until the good news underlying the high expected rate of return were announced, at which point the stock price would rise abruptly. If it were widely known that the expected rate of return was high, then many traders would buy the stock, forcing the price to rise abruptly until the future expected rate of return was reduced to the appropriate risk-adjusted level. These abrupt rises in price would be captured econometrically as volatility, and not as autocorrelation. Thus, if we see autocorrelation in stock returns after eliminating NT and BAB, it can only come from two sources: TVRP, or the strategic decision of a small group of informed traders to exercise their informational advantage slowly. In short, it must either be TVRP or PPA.

To the best of our knowledge, no paper has asserted that TVRP is a significant source of autocorrelation in daily returns of individual stocks or portfolios over periods of length two years.[22] However, our goal is to develop methods for decomposing return autocorrelation in a general setting, where one might consider weekly, monthly, quarterly, or annual returns (the return period), and where the time horizon might range up to several decades. In this Appendix, we develop general bounds on the bias induced by TVRP. We show that the bias is negligible for daily returns over a time horizon of two years, but it may need to be taken into account in other settings.

The analysis shows that the bias in the measured autocorrelation resulting from TVRP depends on both the return period and the time horizon. The bias becomes larger as the time horizon increases (because the variation of return resulting from TVRP is larger over longer time horizons) and larger as the return period increases (daily returns are much noisier than yearly returns, so the bias represents a larger fraction of the total return variance). We find that the bias in daily returns over a two-year time horizon is very small; however, the bias in annual returns over a horizon of decades could be substantial.

All of our correlations are calculated over two-year subperiods of the period 1993-2002. How much might the expected daily risk-adjusted rate of return of a well-diversified stock portfolio vary over one of these two-year subperiods? Stocks and portfolios could conceivably have expected rates of return below the risk-free rate, but only if they were negatively correlated with undiversifiable risks and thus provided insurance against those risks. Since broadly diversified stock portfolios are positively correlated with two important undiversifiable risks (the market as a whole and aggregate income), it is implausible that investors would hold the portfolio if it had an expected rate of return below the risk-free rate. To maintain equilibrium, stock prices would have to fall to raise the future expected rate of return sufficiently to induce stockholders to retain their holdings. On the other hand, if the expected rate of return of the portfolio exceeded the risk-free rate by 15% per annum, investors would surely choose to substantially increase their stockholdings: taking the volatility into account, there is very little chance of a substantial decline in stock prices and a very good chance of a substantial gain. Over the two-year periods we consider, the average variation of the risk-free rate (as measured by the three-month Treasury Bill Rate) is 2.49%.[23] Thus, we assume that TVRP will induce variation in the return on a well-diversified portfolio of no more than 18% per annum over a two-year period.

In the case of an individual stock, the expected rate of return should reflect the risk premia of the factors underlying its pricing. Some stocks may have low—even negative—risk premia, while others may have large risk premia. However, the correlation of any given stock with the main risk factors should be relatively stable over time periods of two years.[24] Thus, we assume that TVRP induce a variation of no more than 18% per annum in the expected rate of return of each of our securities, and each of the portfolios we consider, in any of our two-year periods. This assumption does not restrict the variation in expected rates of return across securities; our assumption limits only the variation across time for a given security.

We now turn to the effect of TVRP on our correlation estimates. For the sake of simplicity, we assume that securities prices follow Geometric Itô Processes; we believe similar estimates would hold for Geometric Lévy Processes. Suppose the security (or portfolio) price S follows the stochastic differential equation

$$\frac{dS}{S} = \mu\, dt + \sigma\, dW \tag{13}$$

where W is a standard Wiener Process, and μ and σ are continuous deterministic functions of time. Assume there are 250 trading days per year. Let

$$\begin{aligned}
\sigma_k &= \sigma\left(\frac{k}{250}\right) && (14)\\
\mu_k &= \mu\left(\frac{k}{250}\right) && (15)\\
\bar{\mu} &= \frac{1}{498}\left(\frac{\mu_1}{2} + \sum_{k=2}^{498}\mu_k + \frac{\mu_{499}}{2}\right) && (16)\\
&\simeq \frac{1}{2}\int_0^2 \mu(t)\, dt && (17)\\
\Delta W_k &= W\left(\frac{k+1}{250}\right) - W\left(\frac{k}{250}\right) && (18)\\
\bar{v} &= \frac{1}{498}\left(\frac{\sigma_1 \Delta W_1}{2} + \sum_{k=2}^{498}\sigma_k \Delta W_k + \frac{\sigma_{499}\Delta W_{499}}{2}\right) && (19)\\
&\simeq \frac{1}{498}\int_0^2 \sigma\, dW && (20)\\
r_k &= \frac{\mu_k}{250} + \sigma_k \Delta W_k && (21)\\
\bar{r} &= \frac{1}{498}\left(\frac{r_1}{2} + \sum_{k=2}^{498} r_k + \frac{r_{499}}{2}\right) && (22)\\
&= \frac{\bar{\mu}}{250} + \bar{v} && (23)\\
\sigma_r^2 &= \frac{1}{498}\left(\frac{(r_1-\bar{r})^2}{2} + \sum_{k=2}^{498}(r_k-\bar{r})^2 + \frac{(r_1-\bar{r})^2}{2}\right) && (24)
\end{aligned}$$

$$= \frac{1}{498}\left(\frac{1}{2}\left(\frac{\mu_1-\bar{\mu}}{250}+\sigma_1\Delta W_1-\bar{v}\right)^2+\sum_{k=2}^{498}\left(\frac{\mu_k-\bar{\mu}}{250}+\sigma_k\Delta W_k-\bar{v}\right)^2 \right. \tag{25}$$

$$\left. +\frac{1}{2}\left(\frac{\mu_{499}-\bar{\mu}}{250}+\sigma_{499}\Delta W_{499}-\bar{v}\right)^2\right) \tag{26}$$

$$\simeq \frac{1}{2}\int_0^2\left(\frac{\mu(t)-\bar{\mu}}{250}\right)^2 dt+\frac{2}{498}\int_0^2\left(\frac{\mu(t)-\bar{\mu}}{250}\right)\sigma(t)\,dW-\bar{v}\int_0^2\frac{\mu(t)-\bar{\mu}}{250}\,dt \tag{27}$$

$$+\frac{1}{498}\left(\frac{\sigma_1^2(\Delta W_1)^2}{2}+\sum_{k=2}^{498}\sigma_k^2(\Delta W_k)^2+\frac{\sigma_{499}^2(\Delta W_{499})^2}{2}\right)-\bar{v}^2 \tag{28}$$

$$\simeq \frac{1}{2}\int_0^2\left(\frac{\mu(t)-\bar{\mu}}{250}\right)^2 dt+\frac{2}{498}\int_0^2\left(\frac{\mu(t)-\bar{\mu}}{250}\right)\sigma(t)\,dW \tag{29}$$

$$+\frac{1}{498}\int_0^2\sigma^2(t)\,dt-\bar{v}^2 \tag{30}$$

Then the Pearson sample autocorrelation coefficient is given by

$$r_p = \frac{\sum_{k=1}^{498}\left(r_k-\frac{1}{498}\sum_{j=1}^{498}r_j\right)\left(r_{k+1}-\frac{1}{498}\sum_{j=2}^{499}r_j\right)}{\sqrt{\sum_{k=1}^{498}\left(r_k-\frac{1}{498}\sum_{j=1}^{498}r_j\right)^2}\sqrt{\sum_{k=2}^{499}\left(r_k-\frac{2}{499}\sum_{j=2}^{499}r_j\right)^2}} \tag{31}$$

$$\simeq \frac{\sum_{k=1}^{498}(r_k-\bar{r})(r_{k+1}-\bar{r})}{\sqrt{\sum_{k=1}^{498}(r_k-\bar{r})^2}\sqrt{\sum_{k=2}^{499}(r_k-\bar{r})^2}} \tag{32}$$

$$\simeq \frac{\sum_{k=1}^{498}(r_k-\bar{r})(r_{k+1}-\bar{r})}{498\sigma_r^2} \tag{33}$$

$$= \frac{\sum_{k=1}^{498}\left(\frac{\mu_k-\bar{\mu}}{250}+\sigma_k\Delta W_k-\bar{v}\right)\left(\frac{\mu_{k+1}-\bar{\mu}}{250}+\sigma_{k+1}\Delta W_{k+1}-\bar{v}\right)}{498\sigma_r^2} \tag{34}$$

$$\simeq \frac{1}{\sigma_r^2}\left(\frac{1}{2}\int_0^2\left(\frac{\mu(t)-\bar{\mu}}{250}\right)^2 dt+\frac{2}{498}\int_0^2\left(\frac{\mu(t)-\bar{\mu}}{250}\right)\sigma(t)\,dW \right. \tag{35}$$

$$\left. -\bar{v}\int_0^2\left(\frac{\mu(t)-\bar{\mu}}{250}\right)dt+\frac{1}{498}\sum_{k=1}^{498}\sigma_k\sigma_{k+1}\Delta W_k\Delta W_{k+1}-\bar{v}^2\right) \tag{36}$$

$$= \frac{1}{\sigma_r^2}\left(\frac{1}{2}\int_0^2\left(\frac{\mu(t)-\bar{\mu}}{250}\right)^2 dt+\frac{2}{498}\int_0^2\left(\frac{\mu(t)-\bar{\mu}}{250}\right)\sigma(t)\,dW \right. \tag{37}$$

$$\left. +\frac{1}{498}\sum_{k=1}^{498}\sigma_k\sigma_{k+1}\Delta W_k\Delta W_{k+1}-\bar{v}^2\right) \tag{38}$$

$$\simeq \frac{A+Z}{B+Z} \tag{39}$$

where

$$A = \frac{1}{498}\sum_{k=1}^{498}\sigma_k\sigma_{k+1}\Delta W_k\Delta W_{k+1}-\bar{v}^2 \tag{40}$$

$$B = \frac{1}{498}\int_0^2 \sigma(t)^2\,dt - \bar{v}^2 \tag{41}$$

$$Z = Z_1 + Z_2 \tag{42}$$

$$Z_1 = \frac{1}{2}\int_0^2 \left(\frac{\mu(t)-\bar{\mu}}{250}\right)^2 dt \tag{43}$$

$$Z_2 = \frac{2}{498}\int_0^2 \left(\frac{\mu(t)-\bar{\mu}}{250}\right)\sigma(t)\,dW \tag{44}$$

In the absence of TVRP, we would have $\mu(t) = \bar{\mu}$ for all t, hence Z_1 and Z_2 would be identically zero. In the presence of TVRP, Z_1 and Z_2 induce a bias into our measurement of PPA. Notice that neither Z_1 nor Z_2 depends on the rate at which μ changes, only on the distribution of μ and (in the case of Z_2) the correlation between μ and σ. Z_1 is a positive constant. Assuming that μ is distributed uniformly over an interval of length $18\% = .18$ per annum, $Z_1 = 4.320 \times 10^{-8}$. Z_2 is normally distributed with mean zero and standard deviation

$$\sigma_Z = \frac{2}{498}\sqrt{\int_0^2 \left(\frac{\mu(t)-\bar{\mu}}{250}\right)^2 \sigma^2(t)\,dt} \tag{45}$$

Assuming that μ is uniformly distributed over an interval of length .18 per annum and σ is constant, $\sigma_Z = 1.180 \times 10^{-6}\sigma$. Moreover, the conditional distribution of Z_2, conditional on A and B, is asymptotically normal.

Our tests of portfolio autocorrelation (putting aside the tests involving SPDRs) compute the daily conventional and intraday return autocorrelations of portfolios. The rejection of the null hypotheses (conventional return autocorrelation is less than or equal to zero in small, medium and large firm portfolios, intraday return autocorrelation is less than or equal to zero in small and medium firm portfolios) is overwhelming, and it is easy to see that the small bias induced by TVRP—the bias induced by Z—cannot make any difference in those results. However, the effect of the bias on the tests involving individual stock autocorrelations and the tests involving SPDRs requires more careful analysis. Since those tests are based on comparing the actual number of rejections to the expected number of rejections, we need to carefully estimate the effect of the bias on the expected number of rejections.

Notice that A and B are quadratic in σ (i.e. if we double the function $\sigma(t)$ at all times, then A and B are quadrupled), while Z_1 is independent of σ and Z_2 is linear in σ. Thus, the bias induced in r_p by Z is maximized when σ is minimized. The returns on individual stocks in a portfolio are more volatile than the returns of the portfolio, and the returns of smaller stocks are more volatile than the returns of larger stocks. We take the volatility of the S&P 500 index as a lower bound on the volatility of the individual stocks in our analysis. For the S&P 500 index, the average value of σ_r, over our five two-year subperiods, is .01041. Assuming σ is constant, we obtain the estimate $\sigma = \sqrt{249}\ln(1.01041) \simeq .16341$. With probability $2 \times (1 - N(4)) > 1 - 2 \times 10^{-4}$, $\left|B - (\sigma_r)^2\right| = |Z| \leq 4.320 \times 10^{-8} + 4 \times 1.18 \times 10^{-6}\sigma = 8.145 \times 10^{-7}$, so $B = (1.084 \pm .008) \times 10^{-4}$; since the bias is maximized when B is minimized, we assume $B = 1.076 \times 10^{-4}$, $\sigma = .1600$, and $\sigma_Z = 1.888 \times 10^{-7}$.

The Pearson test compares $\frac{\sqrt{498}r_p}{\sqrt{1-r_p^2}}$ to the standard normal. We have

$$\sqrt{498}\frac{r_p}{\sqrt{1-r_p^2}} = \sqrt{498}\frac{\frac{A+Z}{B+Z}}{\sqrt{1-\frac{(A+Z)^2}{(B+Z)^2}}} \tag{46}$$

$$= \sqrt{498}\frac{A+Z}{\sqrt{(B+Z)^2-(A+Z)^2}} \tag{47}$$

Let

$$g_{AB}(Z) = \sqrt{498}\frac{A+Z}{\sqrt{(B+Z)^2-(A+Z)^2}} \tag{48}$$

When $|A| \leq B$ and $|Z| \leq B$, $\frac{A+Z}{B+Z}$ is concave in Z. It follows that on the relevant range of values ($g_{AB}(Z) \simeq 1.96$, so $\frac{A+Z}{B+Z} \simeq .09$), g_{AB} is concave, so letting $h_{AB}(Z)$ be the first-order Taylor series of g_{AB}, we have

$$g_{AB}(Z) \leq h_{AB}(Z) = \sqrt{\frac{498}{B^2-A^2}}\left(A + \frac{B}{B+A}Z\right) \tag{49}$$

on the relevant range of values, and $g_{AB}(0) = h_{AB}(0)$.

The correct test would measure only the autocorrelation coming from PPA. Our actual test measures the autocorrelation coming from PPA and TVRP. Our most important null hypothesis is that the autocorrelation is less than or equal to zero. Since $Z_1 \geq 0$, and $|A| \leq B$, the presence of Z_1 leads our test to reject the null hypothesis in situations in which the correct test would fail to reject; in any situation in which the correct test rejects, our test will also reject. Thus, the presence of Z_1 increases the expected number of rejections.

$|Z_2|$ is typically larger than Z_1. However, we shall see that Z_2 induces a smaller bias in the expected number of rejections because Z_2 can be either positive or negative. The presence of Z_2 leads our test to reject in some cases in which the correct test does not reject, and to fail to reject in some cases in which the correct test does reject. The symmetry of Z_2 will imply that the two effects very nearly cancel.

Since $g_{AB}(Z) \leq h_{AB}(Z)$ on the relevant values, the probability of rejection in our test, which compares $g_{AB}(Z)$ to the standard normal, is lower than the probability of rejection in a hypothetical test comparing $h_{AB}(Z)$ to the standard normal. Noting that A and B are random variables, let $Y = \frac{\sqrt{498}A}{\sqrt{B^2-A^2}}$ be the random variable $h_{AB}(0)$. Let N denote the cumulative distribution function of the standard normal. The probability that Z changes an insignificant value to a significant value, using the critical value α, is the probability that $Y = h_{AB}(0) < \alpha$ and $h_{AB}(Z) \geq \alpha$, which equals

$$\frac{1}{\sqrt{2\pi}\sigma_Z} \int_0^\infty e^{-z^2/2\sigma_Z^2} \left(N(\alpha) - N\left(\alpha - \frac{\sqrt{498}B}{\sqrt{B^2-A^2}(B+A)}(Z_1+z)\right)\right) dz \tag{50}$$

$$\leq \; 1 - N(4) + \frac{1}{\sqrt{2\pi}\sigma_Z} \int_0^{4\sigma_Z} e^{-z^2/2\sigma_Z^2} \left(N(\alpha) - N\left(\alpha - \gamma_1 (Z_1+z)\right)\right) dz \tag{51}$$

where γ_1 is the value of $\frac{\sqrt{498}B}{\sqrt{B^2-A^2}(B+A)}$ corresponding to $Y = \alpha - \gamma_1 (Z_1 + 4\sigma_Z)$. Although it is hard to solve for γ_1 exactly, we can estimate it as follows:

$$\frac{\partial}{\partial A}\left(\frac{\sqrt{498}B}{\sqrt{B^2-A^2}(B+A)}\right) \tag{52}$$

$$= \; \sqrt{498}B\left(\frac{\partial}{\partial A}\left(\left(B^2-A^2\right)^{-1/2}(B+A)^{-1}\right)\right) \tag{53}$$

$$= \; \sqrt{498}B\left(A\left(B^2-A^2\right)^{-3/2}(B+A)^{-1} - \left(B^2-A^2\right)^{-1/2}(B+A)^{-2}\right) \tag{54}$$

$$> \; -\sqrt{498}B\left(B^2-A^2\right)^{-1/2}(B+A)^{-2} \tag{55}$$

$$\geq \quad -\frac{.92\sqrt{498}}{B^2} \tag{56}$$

on the relevant range (which is included in $1 \leq Y \leq \alpha = 1.96$, so $.044B \leq A \leq .09B$). It follows that

$$\gamma_1 \leq \gamma_0 + \frac{.92\sqrt{498}}{B^2}\left(Z_1 + 4\sigma_Z\right) \tag{57}$$

where γ_0 is the value of $\frac{\sqrt{498}B}{\sqrt{B^2-A^2}(B+A)}$ corresponding to $Y = \alpha$; for $\alpha = 1.96$, we have $\gamma_0 \simeq \frac{.9231\sqrt{498}}{B}$.

Similarly, the probability that Z changes an insignificant value to a significant value is bounded below by

$$\frac{1}{\sqrt{2\pi}\sigma_Z}\int_0^{4\sigma_Z} e^{-z^2/2\sigma_Z^2}\left(N\left(\alpha + \gamma_2\left(z - Z_1\right)\right) - N(\alpha)\right)\,dz \tag{58}$$

where

$$\gamma_2 \geq \gamma_0 - \frac{1.01\sqrt{498}}{B^2}(4\sigma_Z - Z_1) \tag{59}$$

Thus,

$$0 \leq \gamma_1 - \gamma_2 \quad \leq \quad \frac{8\sqrt{498}\sigma_Z}{B^2} \tag{60}$$

$$0 \leq \gamma_1 + \gamma_2 \quad \leq \quad 2\gamma_0 \tag{61}$$

$$0 \leq \gamma_1^2 - \gamma_2^2 \quad = \quad (\gamma_1 - \gamma_2)(\gamma_1 + \gamma_2) \tag{62}$$

$$\leq \quad \frac{16\sqrt{498}\sigma_Z}{B^2}\gamma_0 \tag{63}$$

$$0 \leq \gamma_1^2 + \gamma_2^2 \quad \leq \quad (\gamma_1 + \gamma_2)^2 \tag{64}$$

$$\leq \quad 4\gamma_0^2 \tag{65}$$

Using the second order Taylor Expansion for the normal cumulative distibution function, the increase in the probability of rejection resulting from Z is bounded above by

$$\frac{1}{\sqrt{2\pi}\sigma_Z}\int_0^{4\sigma_Z} e^{-z^2/2\sigma_Z^2}\left(2N(\alpha) - N\left(\alpha - \gamma_1\left(Z_1 + z\right)\right) - N\left(\alpha + \gamma_2\left(z - Z_1\right)\right)\right)\,dz \tag{66}$$

$$
\begin{aligned}
& +1-N(4) && (67)\\
\leq\ & \frac{1}{\sqrt{2\pi}\sigma_Z}\int_0^\infty e^{-z^2/2\sigma_Z^2}\left(N'(\alpha)\left(\gamma_1+\gamma_2\right)Z_1+\left(\gamma_1-\gamma_2\right)z\right)\,dz && (68)\\
& -\frac{1}{\sqrt{2\pi}\sigma_Z}\int_0^\infty e^{-z^2/2\sigma_Z^2}\frac{N''(\alpha)}{2}\left(\left(\gamma_1\left(Z_1+z\right)\right)^2+\left(\gamma_2\left(z-Z_1\right)\right)^2\right)\,dz && (69)\\
& +\frac{1}{\sqrt{2\pi}\sigma_Z}\int_0^{4\sigma_Z} e^{-z^2/2\sigma_Z^2}\frac{N'''(\xi(z))}{6}\left(\gamma_1\left(Z_1+z\right)\right)^3\,dz+10^{-4} && (70)\\
& \text{for some measurable function } \xi:[0,\infty)\rightarrow\mathbf{R} && (71)\\
\leq\ & N'(\alpha)\left(\frac{\gamma_1+\gamma_2}{2}Z_1+\frac{(\gamma_1-\gamma_2)\sigma_Z}{\sqrt{2\pi}}\right) && (72)\\
& -\frac{N''(\alpha)}{2}\left(\frac{\gamma_1^2+\gamma_2^2}{2}Z_1^2+\frac{2\sigma_Z}{\sqrt{2\pi}}\left(\gamma_1^2-\gamma_2^2\right)Z_1+\frac{\gamma_1^2+\gamma_2^2}{2}\sigma_Z^2\right) && (73)\\
& +\frac{N'''(\xi(z))\gamma_1^3}{6}\left(\frac{Z_1^3}{2}+\frac{3Z_1^2\sigma_Z}{\sqrt{2\pi}}+\frac{3Z_1\sigma_Z^2}{2}+\frac{2\sigma_Z^3}{\sqrt{2\pi}}\right) && (74)\\
& +10^{-4} && (75)\\
\leq\ & \frac{e^{-\alpha^2/2}}{\sqrt{2\pi}}\left(\gamma_0 Z_1+\frac{8\sqrt{498}\sigma_Z^2}{\sqrt{2\pi}B^2}\right) && (76)\\
& +\frac{\alpha e^{-\alpha^2/2}}{2\sqrt{2\pi}}\left(2\gamma_0^2 Z_1^2+\frac{32\sqrt{498}\sigma_Z^2}{\sqrt{2\pi}B^2}\gamma_0 Z_1+2\gamma_0^2\sigma_Z^2\right) && (77)\\
& +\frac{e^{-3/2}\gamma_1^3}{3\sqrt{2\pi}}\left(\frac{Z_1^3}{2}+\frac{3Z_1^2\sigma_Z}{\sqrt{2\pi}}+\frac{3Z_1\sigma_Z^2}{2}+\frac{2\sigma_Z^3}{\sqrt{2\pi}}\right)+10^{-4} && (78)
\end{aligned}
$$

For the reasons explained above, with probability $1-2\times 10^{-4}$, we may take $\alpha = 1.96$, $\sigma = 0.1600$, $B = 1.076\times 10^{-4}$, $\sigma_z = 1.888\times 10^{-7}$, $Z_1 = 4.320\times 10^{-8}$, $\gamma_0 = \frac{.9231\sqrt{498}}{B} = 1.914\times 10^5$, $\gamma_1 \leq \gamma_0 + \frac{.92\sqrt{498}}{B^2}\left(Z_1+4\sigma_z\right) = 1.929\times 10^5$. Then the increase in the probability of rejection resulting from Z is at most

$$
\begin{aligned}
& \frac{e^{-\alpha^2/2}}{\sqrt{2\pi}}\left(\gamma_0 Z_1+\frac{8\sqrt{498}\sigma_Z^2}{\sqrt{2\pi}B^2}\right) && (79)\\
& +\frac{\alpha e^{-\alpha^2/2}}{2\sqrt{2\pi}}\left(2\gamma_0^2 Z_1^2+\frac{32\sqrt{498}\sigma_Z^2}{\sqrt{2\pi}B^2}\gamma_0 Z_1+2\gamma_0^2\sigma_Z^2\right) && (80)\\
& +\frac{e^{-3/2}\gamma_1^3}{3\sqrt{2\pi}}\left(\frac{Z_1^3}{2}+\frac{3Z_1^2\sigma_Z}{\sqrt{2\pi}}+\frac{3Z_1\sigma_Z^2}{2}+\frac{2\sigma_Z^3}{\sqrt{2\pi}}\right)+3\times 10^{-4} && (81)\\
=\ & .0584\left(8.271\times 10^{-3}+2.193\times 10^{-4}\right) && (82)\\
& +.0573\left(1.368\times 10^{-4}+7.254\times 10^{-6}+2.613\times 10^{-3}\right) && (83)
\end{aligned}
$$

$$+.0297\left(7.174\times 10^{15}\right)\left(4.031\times 10^{-23}+4.217\times 10^{-22}\right. \quad (84)$$

$$\left.+2.310\times 10^{-21}+5.370\times 10^{-21}\right) \quad (85)$$

$$+3\times 10^{-4} \quad (86)$$

$$= 4.958\times 10^{-4}+1.580\times 10^{-4}+1.735\times 10^{-5}+3\times 10^{-4} \quad (87)$$

$$= 9.555\times 10^{-4} \quad (88)$$

Thus, the bias induced by TVRP increases the probability of rejection from .0250 by at most .001 to .026, so the expected number of rejections in 100 autocorrelations increases by at most .1 from 2.5 to 2.6. Our tests of individual stock autocorrelations, as well as our tests using SPDRs, are all based on comparing the number of rejections to 2.5. Changing 2.5 to 2.6 to adjust for the bias increases the p-values slightly but makes no qualitative change in those findings. Since the estimates we have just given very likely substantially overstate the bias, we feel confident in the results reported in the tables.

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Footnotes

[1] The momentum effect has been cited as an explanation of medium-term (3 to 12 months) autocorrelation (see Jegadeesh and Titman (1993)). The momentum effect is properly viewed as a form of PPA. We make no attempt in this paper to model PPA, and thus need not be concerned with the various forms of trader behavior that can give rise to it. Rather, we present methods to decompose return autocorrelation into the various components. In addition, the medium-term momentum effect is of little relevance to daily return autocorrelation, which is the focus of the empirical work reported here.

[2] For example, Ederington and Lee (1995), Kim et al. (1997), Busse and Green (2002), and Adams et al. (2004) established by direct tests that the incorporation of publicly-released information into securities' prices is not instantaneous. Ederington and Lee found lags of not more than 40 seconds in interest rate and foreign exchange futures in response to scheduled macroeconomic news releases; since trade occurs over this 40 second interval, they find PPA. Kim et al. studied price adjustment when favored clients are given access to an analyst's initial buy recommendation prior to the opening of the market: "For NYSE/AMEX stocks, almost all of the private information contained in analysts' recommendations is reflected in the opening trade. There are only minor gains to informed traders subsequent to the initial trade, gains that are typically less than transaction costs. … However, [on the NYSE and AMEX] considerable time elapses, on average 10 minutes, before the specialist sets a market clearing price. Thus, in the specialist market, there is a substantial period during which no market clearing price is found, so there is a loss in liquidity." Note that Kim et al. found a delay in price adjustment, but did *not* find PPA on the NYSE and AMEX, since no trade occurs in the period before the specialist sets the price; they do find PPA on the Nasdaq. Busse and Green found that the prices of NYSE, AMEX, and Nasdaq stocks positively mentioned on the CNBC TV Morning Call and Midday Call segments adjust within one minute; traders who execute within 15 seconds make small but significant profits, indicating that PPA is occurring but for a very brief period. Adams et al. studied the response of portfolios of stocks (sorted by size) to inflation surprises. The return of each portfolio was calculated on horizons of one, five, and fifteen minutes, one hour, and one day; if a stock does not trade within a given horizon, it is excluded from the calculation of return on its portfolio. They found substantial responses within one minute and, indeed, the coefficients at the one-minute horizon exceed those at the one-day horizon in five of their ten

regressions (five portfolios times two measures of inflation). The coefficients at the fifteen-minute horizon are even higher, which Adams et al. interpreted as implying that the full response takes somewhat more than one minute, but less than fifteen minutes. To us, this indicates that some overshooting is occurring at the fifteen-minute horizon. We acknowledge, as Adams et al. pointed out, that the statistical significance is generally lower at the one-day horizon, because additional time allows for additional volatility that raises the standard errors. However, the coefficients at the one-day horizon are still unbiased estimators of the permanent effect of the inflation surprise, so that if the one-minute coefficients roughly agree with the one-day coefficients, while the fifteen-minute coefficients are noticeably higher than the one-day coefficients, this says to us that prices reach the correct level within one minute, then proceed to overshoot over the next fourteen minutes; overshooting should result in negative autocorrelation. Adams et al. estimated that prices of large firms complete their adjustment (which we believe involves some overshooting) within six trades, and within a smaller number of trades for small firms. Significantly for our study, Adams et al. conclude that "differential response times to inflation news cannot be driving daily, weekly and monthly cross-autocorrelation." Thus, we are not aware of any evidence that the slow incorporation of publicly-released information is a factor in daily return autocorrelation. When information is released publicly, there is no opportunity for informed agents to benefit by trading slowly so as to conceal their information, so we expect price adjustment to be rapid. By contrast, when private information is possessed by some agents and *not* released publicly, market microstructure models predict that informed agents may strategically choose to exercise their informational advantage slowly, over several days, and this slow price adjustment has the potential to generate daily return autocorrelation. Because the private information of traders is not observable, one cannot apply the methods of Ederington and Lee; Kim et al.; Busse and Green; and Adams et al. to the incorporation of private information into prices. In this paper, we devise direct tests of the relative roles of NT, BAB, PPA, and TVRP in stock return autocorrelation.

[3] In a portfolio of stocks, the individual stocks are traded; the portfolio itself is not traded, and its price is obtained by averaging the prices of the individual stocks it contains. Thus, while the price of an individual stock may bounce between the bid and ask, there is no bid or ask between which the portfolio price jumps. If the bounce process, which determines whether a given trade occurs at the bid or ask price, were independent across different stocks, bid-ask bounce would produce a slight negative autocorrelation in portfolio returns coming from the negative

autocorrelation of the individual stocks in the portfolio; the cross bid-ask bounce effects would be zero. In practice, the bounce process probably shows positive correlation across stocks; if stock prices generally rise (fall) just before the close, then most stocks final trade will be at the ask (bid) price, inducing negative autocorrelation in the daily portfolio return. Thus, bid-ask bounce should cancel some of the positive autocorrelation in daily portfolio returns that results from NT, PPA, and TVRP.

[4] Inventory costs should lead specialists to make transitory adjustments in prices in order to bring their inventories back to the desired level (see Lyons (2001), pages 130-133). Because the mean reversion of specialists' inventories has quite a long half-life (Madhavan and Smidt (1993)), it seems unlikely that inventory costs are a significant source of *daily* return autocorrelation. Although the autocorrelation resulting from inventory costs is often described as a microstructure *bias*, we see it as a form of PPA. The specialist deliberately adjusts the stock price above (below) the price that equates traders' buy and sell orders in order to increase (reduce) his inventory to the desired level, then gradually lowers (raises) the price, even though the inventory imbalance conveyed no information. The autocorrelation results from the slow decay of the adjustment.

[5] However, the application of the methods of this paper to monthly, quarterly, or annual return periods, or to time horizons substantially longer than two years, may require adjustment for TVRP.

[6] Garcia Blandon (2001) examines the return autocorrelation of the IBEX-35, an index composed of the 35 most liquid Spanish companies. He computes returns on an open-to-close basis. It appears he takes the opening price to be the index value when the market opens, rather than the average of the opening prices of the stocks comprising the index; since some of the stocks in the index will not trade at the market opening, the opening price of the index will involve some stale prices, so NT will not be completely eliminated. Garcia Blandon finds that the autocorrelation disappears when the index returns are computed on an open-to-close basis; this is analogous to our finding for large firms, but contrasts with our finding for small and medium firms.

[7] More precisely, the absence of PPA and TVRP imply the stated conclusion. See Appendix B for a detailed analysis of the magnitude of the potential bias resulting from TVRP.

[8] Since there is no trade in the stock after time t_i, the intraday return also equals $r_{s_i ti}$, where t is the time when the market closes.

[9] The assumption in Roll's model that the coin tosses are independent across trades is restrictive. Choi et al. (1988) showed that serial correlation of either sign in the coin tosses affects the magnitude, but not the sign, of the autocorrelation in conventional daily returns induced by bid-ask bounce. Positive (negative) serial correlation in the coin tosses of a given stock induces negative (positive) autocorrelation of intraday returns, but it appears that the magnitude is much smaller than that of the autocorrelation of conventional daily returns. It seems likely the serial correlation of the coin tosses is positive, so we expect bid-ask bounce to induce slight negative autocorrelation of individual stock intraday returns. If we extend Roll's model to multiples stocks, and assume that the coin tosses are independent *across stocks*, the cross-autocorrelations induced by bid-ask bounce will be zero. It is unclear how restrictive the assumption of independence of the coin tosses across stocks is. If the coin tosses are correlated across stocks, it appears that the correlation should be positive: if the market as a whole is rising, this seems likely to cause buyers to raise their bids to match the current ask; if the market as a whole is falling, this seems likely to cause sellers to lower their asks to match the current bid. Positive correlation of the coin tosses across stocks would result in negative cross-autocorrelation in daily returns, and slight negative cross-autocorrelation in intraday returns.

[10] The specific tests for autocorrelation will be described in Section 2.3.

[11] The reader might have expected us to set the intraday return of that stock to be zero for that day. Doing so could introduce an NT bias for essentially the same reason that imputing a zero return on days on which a given stock does not trade induces negative autocorrelation in individual daily stock returns. The results when the observations are included and set to zero are essentially the same.

[12] Boudoukh et al. (1994) report first-order autocorrelation of 0.23 for weekly returns of an equally-weighted index and 0.36 for weekly returns of a small-stock portfolio.

[13] Using weekly data, Connolly and Stivers (2003) "find substantial momentum (reversals) in consecutive weekly returns when the latter week has unexpectedly high (low) turnover." In contrast, Chordia and Swaminathan (2000) use turnover and return shocks for their tests, using a model specification similar to those of Campbell et al. (1993) and Llorente et al. (2002). Even though the model specification of Connelly and Stivers differs from that of Chordia and Swaminathan, the results from Connolly and Stivers are along the same line as those of Chordia and Swaminathan; and Llorente et al.; supporting the partial price adjustment hypothesis. For other literature on the

PPA hypothesis, see Brennan et al. (1993), Mech (1993), Badrinath et al. (1995), McQueen et al. (1996), among others.

14 As above, the reader might have expected us to set the return to zero on days on which the stock does not trade. We chose instead to omit the data for the reasons explained above. Setting the return to zero and including it in the data makes little difference in the results.

15 Note that this does not imply that the stock will trade within two or three minutes of a move of the SPDRs; rather, it indicates that any trade occurring at least three minutes after the SPDR move should be at the correct price and should not exhibit PPA.

16 See Sheskin (1997), page 633.

17 The analysis began in the fall of 2004. SPDRs were not introduced until 1993, so we could not go back before 1993. We did not have access to complete data for 2004 at that time, so we could not use six two-year periods 1993-2004. Thus, we chose to use five two-year periods 1993-2002. At the time, we paid no attention to the presence of the bubble in our data set.

18 This method is suggested in Lee et al. (1993). To test the robustness of our results, we also used a twenty percent criterion, but we found no significant difference between the two rules.

19 For example, if the TAQ dataset reported successive trades in a stock at prices of \$10, \$41, \$11, we would eliminate the transaction with a reported price of \$41.

20 Kadlec and Patterson (1999) report that the average small stock trades within 3 hours of the close and the average large stock trades within 2 minutes of the close on each day.

21 Small (medium) firms exhibit five (four) rejections out of five at the 0.5% (one-sided) level; the probability of five out of five rejections is $(.005)^5 < 10^{-11}$, while the probability of four out of five rejections is $5(.005)^4(.995) < 10^{-8}$.

22 Conrad and Kaul (1988, 1989) and Conrad et al. (1991) (hereafter collectively abbreviated as CKN) estimate that predictable time-varying rates of return can explain 25% of the variance in weekly and monthly portfolio returns. They do not apply their methodology to daily returns; if they had, they presumably would have found a somewhat smaller percentage. As noted in the text, predictable time-varying rates of return are simply autocorrelation by

another name, and are not necessarily attributable to TVRP. CKN invoke a strong form of the Efficient Markets Hypothesis to assert that, since anyone could in principle exploit any knowledge of the TVRP, there cannot be any exploitable information. Since testing for PPA is, in effect, testing a version of the Efficient Markets Hypothesis, we are unwilling to impose the Efficient Markets Hypothesis as an assumption. The predictable expected rates of return documented by CKN vary substantially from week to week, and we find it implausible that TVRP vary this much over the span of a week or two; see Ahn et al. (2002, page 656), who note that "time variation in [risk premia] is not a high-frequency phenomenon: asset pricing models link expected returns with changing investment opportunities, which, by nature, are low-frequency events" (the original says "expected returns," but it is clear from the context that by this, they meant risk premia as we use the terms in this paper).

[23] The variations (max-min) in the three-month Treasury Bill rates for our five two-year subperiods are as follows: 3.34% (6.39%-3.05%) (93-94), 1.28% (6.40%-5.12%) (95-96), 0.86% (5.83%-4.97%) (97-98), 1.99% (6.84%-4.85%) (99-00), and 4.96% (6.27%-1.31%) (01-02). The average of the subperiod variations is 2.49%.

[24] The most likely reason for a major change in the correlation of a stock with the risk factors is diversification into a new line of business, or the sale or spin-off of a line of business. In order to significantly change the correlations, the divested or acquired line of business would have to be a reasonably large fraction of the business of the firms as a whole. We eliminate in each data period any firm for which the number of outstanding shares changes by more than 10% in that data period. This should eliminate most firms that acquire substantial new business lines through acquisition, and many of those that divest substantial business lines through sale or spin-off.

Table 1
Descriptive statistics of data

	Small firm	Medium firm	Large firm	SPDRs
Number of firms	100	100	100	-
Market capitalization (in mil. of dollars)				
Max	346.9	1,225.3	475,003.2	-
Mean	135.9	755.9	26,304.6	-
Min	23.9	454.1	4,716.8	-
Daily *portfolio* returns¶				
Mean (%)	0.0604	0.0395	0.0348	0.0004
Std. dev. (%)	0.6273	0.8251	0.8990	0.0108
Average daily trading volume (in shares)	22,200.3	105,637.4	1,458,878.2	42,603.8
Average time interval between the closing trade of individual stock and the closing trade of SPDRs (in seconds)	-2,839.4	-682.5	-73.5	-
Average number of days on which trade occurs	490.6	503.6	504.0	500.2†
First-order autocorrelation of SPDRs				
Conventional daily returns	-	-	-	-0.0199
(Standard error)				(0.0558)

This Table provides descriptive statistics for our 300 NYSE-listed sample firms, stratified in three groups by market capitalization, and for the SPDRs. We sampled the firms every two-year, five times over the ten years spanned from 1993 to 2002. The Table presents the number of firms, the market capitalization (in millions of dollars; max, mean, and min), daily portfolio returns (mean and standard deviation), average daily trading volume (in shares), average time interval between the closing trade of the individual stock and the closing trade of the SPDRs (in seconds), and average number of days on which trade occurs. All statistics except max and min of market capitalization are averages over five two-year intervals. Max and min of market capitalization denote max of max and min of min. The closing trade of an individual stock is the last trade occurring before 4:00 p.m.; the closing trade of the SPDRs is the first trade reported after 4:00 p.m., except on 26 days immediately before holidays on which the market closed early, where we take the last trade occurring before 4:00 p.m. We also report the first-order autocorrelation of SPDRs over ten years using conventional daily returns. All daily returns are calculated in the conventional way: the price at the closing trade on day *d*, minus the price at the last trade prior to day *d*, divided by the price at the last trade prior to day *d*. We use transaction data from TAQ database over the sample period from January 4, 1993 to December 31, 2002. ¶ denotes the statistics of portfolios, not average of individual firms of each group. † denotes that SPDRs introduced to the AMEX on January 29, so that its trading day are short of 19 trading days compared to other stocks.

Table 2
Average *individual* daily return autocorrelations: conventional daily returns
(Null Hypotheses I, IA, IB)

Portfolio	Number of firms	Average *individual* daily returns autocorrelations: conventional daily returns					
		93-94	95-96	97-98	99-00	01-02	1993-2002
Small firm	100	0.0084	0.0034	0.0664**	0.0412**	0.0330**	0.0335**
		(0.0102)	(0.0112)	(0.0093)	(0.0085)	(0.0098)	(0.0043)
Med. firm	100	0.0173*	-0.0058	0.0208*	0.0119	-0.0354++	-0.0009
		(0.0082)	(0.0076)	(0.0097)	(0.0080)	(0.0075)	(0.0036)
Large firm	100	0.0115*	-0.0011	-0.0082	0.0217**	-0.0076	0.0026
		(0.0058)	(0.0058)	(0.0057)	(0.0061)	(0.0055)	(0.0026)

This Table reports the average *individual* daily returns autocorrelations within each firm size group using conventional daily returns. The conventional daily return of each individual stock on day *d* is computed in the usual way: the price at the final trade on day *d*, minus the price at the final trade on day *d*-1, divided by the price at the final trade on day *d*-1. Our sample consists of 300 NYSE-listed sample firms, stratified into the three groups. We use transaction data from the TAQ database over the sample period from January 4, 1993 to December 31, 2002. Numbers in parenthesis are standard errors. In each time-horizon subperiod, ** and * denote positive significance at the 1% and 5% (two-sided) levels, respectively; ++ denotes negative significance at the 1% level. Null Hypothesis IA (IB) is rejected at the 1% level if the autocorrelation is statistically positive (negative) in two or more of the five subperiods; thus, Null Hypothesis IA is rejected among all three firm-size groups. We also report the average autocorrelation within each size group over the entire ten-year data period.

Table 3
Autocorrelation of daily *individual* stock returns: conventional daily returns
(Null Hypotheses II, IIA, IIB)

Portfolio		Autocorrelation of daily *individual* stock returns: conventional daily returns									
		93-94	95-96	97-98	99-00	01-02	1993-2002				
							X_1	p_1	X_2	p_2	p_3
Panel A: Pearson correlation test											
Small firm:	+	23	22	42	30	26	22	0.0000	23	0.0006	*0.0000***
	-	16	23	5	7	10	5	0.0313	7	0.0581	*0.0625*
	+-	39	45	47	37	36	36	0.0001	37	0.0015	*0.0001***
Med. firm:	+	19	12	22	16	4	4	0.0954	12	0.0078	*0.0157**
	-	12	17	11	7	22	7	0.0058	11	0.0109	*0.0116**
	+-	31	29	33	23	26	23	0.0005	26	0.0058	*0.0010***
Large firm:	+	7	4	6	15	1	1	1.0000	4	0.3815	*0.7629*
	-	4	8	9	5	6	4	0.0954	5	0.1875	*0.1907*
	+-	11	12	15	20	7	7	0.1859	11	0.1358	*0.2717*
Panel B: modified Pearson correlation test											
Small firm:	+	18	16	35	23	23	16	0.0001	18	0.0017	*0.0002***
	-	13	15	5	7	6	5	0.0313	6	0.1005	*0.0625*
	+-	31	31	40	30	29	29	0.0002	30	0.0033	*0.0003***
Med. firm:	+	19	8	19	13	3	3	0.4019	8	0.0358	*0.0072***
	-	9	15	9	4	21	4	0.0954	9	0.0232	*0.0463**
	+-	28	23	28	17	24	17	0.0022	23	0.0092	*0.0044***
Large firm:	+	7	5	5	11	1	1	1.0000	5	0.1875	*0.3750*
	-	3	5	6	2	6	2	1.0000	3	0.8038	*1.0000*
	+-	10	10	11	13	7	7	0.1859	10	0.1875	*0.3719*
Panel C: Kendall tau test											
Small firm:	+	9	16	36	26	26	9	0.0017	16	0.0026	*0.0033***
	-	25	32	13	9	11	9	0.0017	11	0.0109	*0.0033***
	+-	34	48	49	35	37	34	0.0001	35	0.0018	*0.0001***
Med. firm:	+	20	10	23	15	3	3	0.4019	10	0.0156	*0.0313**
	-	17	26	12	6	27	6	0.0126	12	0.0078	*0.0157**
	+-	37	36	35	21	30	21	0.0008	30	0.0033	*0.0015***
Large firm:	+	5	4	5	6	5	4	0.0954	5	0.1875	*0.1907*
	-	3	7	2	2	10	2	1.0000	2	1.0000	*1.0000*
	+-	8	11	7	8	15	7	0.1859	8	0.3815	*0.3719*

This Table reports the results of our tests of individual-stock daily return autocorrelations using conventional daily returns. The conventional daily return of each individual stock on day *d* is computed in the usual way: the price at the final trade on day *d*, minus the price at the final trade on day *d*-1, divided by the price at the final trade on day *d*-1. Our sample consists of 300 NYSE-listed sample firms, stratified into the three groups. We use three methods to test whether the correlation is zero: the Pearson correlation test (Panel A), a modified Pearson correlation test (Panel B), and the Kendall tau test (Panel C). Our modified Pearson test uses Andrew's (1991) heteroskedasticity and autoregression consistent (HAC) covariance estimator for the estimation and test of the correlation coefficient. We obtain the Andrew's HAC covariance using the quadratic spectral (QS) kernel with automatic bandwidth selection method. We use transaction data from the TAQ database over the sample period from January 4, 1993 to December 31, 2002. + and - denote the numbers of stocks with statistically significant positive and negative autocorrelation, at the 2.5% level; +-

denotes the numbers of stocks with statistically significant autocorrelation in a two-sided test, at the 5% level. X_1 denotes the first order statistic (minimum) of the observations for the five two time-horizon subperiods, while X_2 denotes the second order statistic (second smallest). p_1 and p_2 denote the probability that X_1 and X_2, respectively, would exceed the observed value in a nonparametric test using only the fact that the numbers in + and – are nonnegative random variables with expectation 2.5. $p_3 = 2 \min\{p_1, p_2\}$ is an upper bound on the probability that either X_1 or X_2 would exceed the observed value. ** and * denote significance at the 1% and 5% level using p_3.

Table 4
Average *individual* daily returns autocorrelations: intraday returns
(Null Hypotheses III, IIIA, IIIB)

Portfolio	Number of firms	Average *individual* daily returns autocorrelations: intraday returns					
		93-94	95-96	97-98	99-00	01-02	1993-2002
Small firm	100	0.0389**	0.0394**	0.0729**	0.0441**	0.0240**	0.0435**
		(0.0074)	(0.0084)	(0.0090)	(0.0083)	(0.0092)	(0.0038)
Med. firm	100	0.0543**	0.0321**	0.0326**	0.0265**	-0.0376^{++}	0.0218**
		(0.0071)	(0.0065)	(0.0096)	(0.0067)	(0.0073)	(0.0032)
Large firm	100	0.0064	-0.0133^{+}	-0.0324^{++}	0.0126*	-0.0103	-0.0064^{++}
		(0.0062)	(0.0063)	(0.0066)	(0.0058)	(0.0054)	(0.0027)

This Table reports the average *individual* daily returns autocorrelations using intraday returns. The intraday return on day *d* of each stock in the portfolio is defined as the price at the final trade on day *d*, minus the price at the first trade on day *d*, divided by the price at the first trade on day *d*. Our sample consists of 300 NYSE-listed sample firms, stratified into the three groups. We use transaction data from the TAQ database over the sample period from January 4, 1993 to December 31, 2002. Numbers in parenthesis are standard errors. Within each of the five two-year time-horizon subperiods, ** and * denote positive significance at the 1% and 5% (two-sided) levels, respectively; $^{++}$ and $^{+}$ denote negative significance. Null Hypotheses IIIA (IIIB) is rejected at the 1% level if the autocorrelation is statistically positive (negative) in two or more of the five two-year subperiods; thus, Null Hypothesis IIIA is overwhelmingly rejected for small and medium firms, while Null Hypothesis IIIB is rejected at the 1% level among large firms. We also report the average autocorrelation within each size group over the entire ten-year data period; these are overwhelmingly statistically positive among small and medium firms, and statistically negative at the 1% level among large firms.

Table 5
Autocorrelation of daily *individual* stock returns: intraday returns
(Null Hypotheses IV, IVA, IVB)

Portfolio		Autocorrelation of daily *individual* stock returns: intraday returns									
		93-94	95-96	97-98	99-00	01-02	1993-2002				
							X_1	p_1	X_2	p_2	p_3
Panel A: Pearson correlation test											
Small firm:	+	28	28	41	32	23	23	0.0000	28	0.0003	*0.0000***
	-	3	7	4	8	11	3	0.4019	4	0.3815	*0.7629*
	+-	31	35	45	40	34	31	0.0001	34	0.0021	*0.0002***
Med. firm:	+	36	23	27	16	2	2	1.0000	16	0.0026	*0.0052***
	-	3	4	10	5	23	3	0.4019	4	0.3815	*0.7629*
	+-	39	27	37	21	25	21	0.0008	25	0.0067	*0.0015***
Large firm:	+	11	7	6	11	3	3	0.4019	6	0.1005	*0.2009*
	-	4	12	20	6	5	4	0.0954	5	0.1875	*0.1907*
	+-	15	19	26	17	8	8	0.0954	15	0.0453	*0.0905*
Panel B: modified Pearson correlation test											
Small firm:	+	24	25	33	23	21	21	0.0000	23	0.0006	*0.0000***
	-	1	5	3	2	8	1	1.0000	2	1.0000	*1.0000*
	+-	25	30	36	25	29	25	0.0003	25	0.0067	*0.0006***
Med. firm:	+	33	21	21	13	3	3	0.4019	13	0.0058	*0.0116**
	-	2	3	7	3	19	2	1.0000	3	0.8038	*1.0000*
	+-	35	24	28	16	22	16	0.0030	22	0.0109	*0.0060***
Large firm:	+	7	5	4	9	1	1	1.0000	4	0.3815	*0.7629*
	-	4	8	14	3	3	3	0.4019	3	0.8038	*0.8038*
	+-	11	13	18	12	4	4	1.0000	11	0.1358	*0.2716*
Panel C: Kendall tau test											
Small firm:	+	28	27	40	39	25	25	0.0000	27	0.0003	*0.0000***
	-	4	5	1	2	10	1	1.0000	2	1.0000	*1.0000*
	+-	32	32	41	41	35	32	0.0001	32	0.0026	*0.0002***
Med. firm:	+	43	24	28	20	4	4	0.0954	20	0.0011	*0.0022***
	-	2	3	6	3	24	2	1.0000	3	0.8038	*1.0000*
	+-	45	27	34	23	28	23	0.0005	27	0.0050	*0.0010***
Large firm:	+	11	5	5	5	3	3	0.4019	5	0.1875	*0.3750*
	-	3	5	8	1	8	1	1.0000	3	0.8038	*1.0000*
	+-	14	10	13	6	11	6	0.4019	10	0.1875	*0.3750*

This Table reports the results of our tests of individual-stock daily return autocorrelations using intraday returns. The intraday return on day *d* of each stock in the portfolio is defined as the price at the final trade on day *d*, minus the price at the first trade on day *d*, divided by the price at the first trade on day *d*. Our sample consists of 300 NYSE-listed sample firms, stratified into the three groups. We use three methods to test whether the correlation is zero: the Pearson correlation test (Panel A), a modified Pearson correlation test (Panel B), and the Kendall tau test (Panel C). Our modified Pearson test uses Andrew's (1991) heteroskedasticity and autoregression consistent (HAC) covariance estimator for the estimation and test of the correlation coefficient. We obtain the Andrew's HAC covariance using the quadratic spectral (QS) kernel with automatic bandwidth selection method. We use transaction data from the TAQ database over the sample period from January 4, 1993 to December 31, 2002. + and - denote the numbers of stocks with statistically significant positive and negative autocorrelation, at the 2.5% level; +- denotes the numbers of stocks with statistically significant autocorrelation in a two-sided test, at the 5% level. X_1 denotes the first order statistic (minimum) of the observations for the five two time-horizon subperiods, while X_2 denotes the second order statistic (second smallest). p_1 and p_2 denote the probability that X_1 and X_2, respectively, would exceed the observed value in a nonparametric test using only the fact that the numbers in + and – are nonnegative random variables with expectation 2.5. $p_3 = 2 \min\{p_1, p_2\}$ is an upper bound on the probability that either X_1 or X_2 would exceed the observed value. ** and * denote significance at the 1% and 5% level using p_3.

Table 6
First-order autocorrelation of daily *portfolio* returns: conventional daily returns

Portfolio	Number of firms	First-order autocorrelation of daily *portfolio* returns: conventional daily returns					
		93-94	95-96	97-98	99-00	01-02	1993-2002
Panel A: Pearson correlation test							
Small firm	100	0.2722**	0.2299**	0.3854**	0.2750**	0.1109*	*0.2550***
Med. firm	100	0.2244**	0.1825**	0.2353**	0.1677**	0.0136	*0.1651***
Large firm	100	0.0441	0.1009*	0.0057	0.1060*	0.0219	*0.0558***
(Std. error)		(0.0445)	(0.0445)	(0.0445)	(0.0446)	(0.0448)	(*0.0199*)
Panel B: modified Pearson correlation test							
Small firm	100	0.2881**	0.2514**	0.3977**	0.2736**	0.1333	*0.2893***
		(0.0499)	(0.0397)	(0.0422)	(0.0435)	(0.0745)	(*0.0208*)
Med. firm	100	0.2250**	0.1888**	0.2321**	0.1618**	0.0165	*0.1664***
		(0.0536)	(0.0478)	(0.0647)	(0.0454)	(0.0586)	(*0.0236*)
Large firm	100	0.0426	0.1130*	-0.0074	0.1006	0.0237	*0.0550***
		(0.0412)	(0.0469)	(0.0540)	(0.0533)	(0.0476)	(*0.0214*)
Panel C: Kendall tau test							
Small firm	100	0.1770**	0.1434**	0.2522**	0.1951**	0.0472	*0.1632***
Med. firm	100	0.1360**	0.1384**	0.1853**	0.0997**	0.0128	*0.1147***
Large firm	100	0.0264	0.0561	0.0387	0.0352	0.0102	*0.0334***
(Std. error)		(0.0298)	(0.0298)	(0.0298)	(0.0299)	(0.0300)	(*0.0134*)

This Table reports the first-order autocorrelation of conventional daily *portfolio* returns using conventional daily returns. The conventional daily return on each stock on day d is defined as the closing price (the price at the final trade of the day) on day d, less the closing price on day $d-1$, divided by the closing price on day $d-1$. The conventional daily return of the portfolio on day d is an equally-weighted average of the conventional daily returns of the stocks in the portfolio, omitting stocks that do not trade on day d. Our sample consists of 300 NYSE-listed sample firms, stratified into the three groups. We use three methods to test whether the correlation is zero: the Pearson correlation test (Panel A), a modified Pearson correlation test (Panel B), and the Kendall tau test (Panel C). Our modified Pearson test uses Andrews' (1991) heteroskedasticity and autoregression consistent (HAC) covariance estimator for the estimation and test of the correlation coefficient. We obtain Andrews' HAC covariance using the quadratic spectral (QS) kernel with automatic bandwidth selection method. We use transaction data from the TAQ database over the sample period from January 4, 1993 to December 31, 2002. Numbers in parenthesis are standard errors. ** and * denote significance at the 1% and 5% level, respectively.

Table 7
First-order autocorrelation of daily *portfolio* returns: intraday returns
(Null Hypothesis V)

Portfolio	Number of firms	First-order autocorrelation of daily *portfolio* returns: intraday returns					
		93-94	95-96	97-98	99-00	01-02	1993-2002
Panel A: Pearson correlation test							
Small firm	100	0.2046**	0.2118**	0.3623**	0.2541**	0.0579	*0.2184***
Med. firm	100	0.1643**	0.2031**	0.2117**	0.1781**	-0.0009	*0.1515***
Large firm	100	-0.0616	-0.0384	-0.1072+	0.0921*	0.0055	*-0.0220*
(Std. error)		(0.0445)	(0.0445)	(0.0445)	(0.0446)	(0.0447)	(*0.0199*)
Panel B: modified Pearson correlation test							
Small firm	100	0.2274**	0.2749**	0.3696**	0.2513**	0.0790	*0.2555***
		(0.0551)	(0.0386)	(0.0611)	(0.0462)	(0.0738)	(*0.0228*)
Med. firm	100	0.1617**	0.2217**	0.2127**	0.1821**	0.0020	*0.1534***
		(0.0513)	(0.0478)	(0.0874)	(0.0480)	(0.0533)	(*0.0240*)
Large firm	100	-0.0608	-0.0284	-0.1017	0.0857	0.0054	*-0.0187*
		(0.0402)	(0.0501)	(0.0714)	(0.0543)	(0.0473)	(*0.0224*)
Panel C: Kendall tau test							
Small firm	100	0.1471**	0.1209**	0.2561**	0.1917**	0.0143	*0.1462***
Med. firm	100	0.0864**	0.1513**	0.1896**	0.0904**	0.0070	*0.1051***
Large firm	100	-0.0434	-0.0197	-0.0333	0.0267	0.0077	*-0.0124*
(Std. error)		(0.0298)	(0.0298)	(0.0298)	(0.0298)	(0.0299)	(*0.0133*)

This Table reports the first-order autocorrelation of daily *portfolio* returns using intraday returns. The intraday return on day *d* of each stock in the portfolio is defined as the price at the final trade on day *d*, minus the price at the first trade on day *d*, divided by the price at the first trade on day *d*; the intraday return of the portfolio is defined as the equally-weighted average of the intraday returns of the stocks in the portfolio, omitting any stocks that do not trade at least twice on the day. Our sample consists of 300 NYSE-listed sample firms, stratified into the three groups. We use three methods to test whether the correlation is zero: the Pearson correlation test (Panel A), a modified Pearson correlation test (Panel B), and the Kendall tau test (Panel C). Our modified Pearson test uses Andrew's (1991) heteroskedasticity and autoregression consistent (HAC) covariance estimator for the estimation and test of the correlation coefficient. We obtain the Andrew's HAC covariance using the quadratic spectral (QS) kernel with automatic bandwidth selection method. We use transaction data from the TAQ database over the sample period from January 4, 1993 to December 31, 2002. Numbers in parenthesis are standard errors. ** and * denote positive significance at the 1% and 5% level, respectively; + denotes negative significance at the 5% level.

Table 8
Proportion of PPA in the autocorrelation of the portfolio returns

Portfolio	93-94	95-96	97-98	99-00	01-02
Panel A: standard deviation					
Small firm: conventional daily return (%)	0.5008	0.4544	0.6232	0.5793	0.9789
Intraday return (%)	0.3950	0.3549	0.4963	0.4879	0.8920
Med. firm: conventional daily return (%)	0.4757	0.4706	0.9388	0.9102	1.3303
Intraday return (%)	0.4146	0.3971	0.8099	0.8418	1.2184
Large firm: conventional daily return (%)	0.5498	0.5845	0.9368	1.0846	1.3394
Intraday return (%)	0.4831	0.5114	0.8354	0.9310	1.1693
Panel B: autocorrelation					
Small firm: conventional daily return	0.2722	0.2299	0.3854	0.2750	0.1109
Intraday return	0.2046	0.2118	0.3623	0.2541	0.0589
Med. firm: conventional daily return	0.2244	0.1825	0.2353	0.1677	0.0135
Intraday return	0.1643	0.2031	0.2117	0.1781	-0.0009
Large firm: conventional daily return	0.0441	0.1009	0.0057	0.1060	0.0219
Intraday return	-0.0616	-0.0384	-0.1072	0.0921	0.0055
Panel C: autocovariance (autocorrelation times variance)					
Small firm: conventional daily return (%)	0.0683	0.0475	0.1497	0.0923	0.1063
Intraday return (%)	0.0319	0.0267	0.0892	0.0605	0.0476
Residual (%)	0.0363	0.0208	0.0604	0.0318	0.0587
Intraday return autocovariance as percentage of intraday plus residual return autocovariance	46.76	56.20	59.62	65.54	44.77
Med. firm: conventional daily return (%)	0.0508	0.0404	0.2074	0.1389	0.0239
Intraday return (%)	0.0282	0.0320	0.1389	0.1262	-0.0013
Residual (%)	0.0225	0.0084	0.0685	0.0127	0.0252
Intraday return autocovariance as percentage of intraday plus residual return autocovariance	55.62	79.24	66.96	90.84	5.03
Large firm: conventional daily return (%)	0.0133	0.0345	0.0050	0.1247	0.0393
Intraday return (%)	-0.0144	-0.0100	-0.0748	0.0798	0.0075
Residual (%)	0.0277	0.0445	0.0798	0.0499	0.0318
Intraday return autocovariance as percentage of conventional daily return autocovariance	34.16	18.41	48.38	64.02	19.14

This Table presents the standard deviation, autocorrelation, autocorrelation weighted by ratio of standard deviation, and autocovariance of conventional and intraday returns in each of the small-, medium-, and large-firm portfolios. The conventional daily return of each individual stock on day *d* is computed in the usual way: the price at the final trade on day *d*, minus the previous closing price, divided by the previous closing price. Our sample consists of 300 NYSE-listed sample firms, stratified into three groups by firm size. We use transaction data from the TAQ database over the sample period from January 4, 1993 to December 31, 2002. The intraday return of each individual stock on day *d* is defined in the following way: the price at the final trade on day *d*, minus the price at the first trade on day *d*, divided by the price at the first trade on day *d*. Our sample consists of 300 NYSE-listed sample firms, stratified into the three groups. We use transaction data from the TAQ database over the sample period from January 4, 1993 to December 31, 2002.

Table 9
ETFs
(Null Hypotheses VI, VIA, VIB)

Portfolio		Correlation of daily *individual* stock returns and SPDRs									
		93-94	95-96	97-98	99-00	01-02	X_1	p_1	X_2	p_2	p_3
Panel A: Pearson correlation test											
Small firm:	+	17	21	41	15	17	15	0.0001	17	0.0021	*0.0003***
	-	0	0	1	0	10	0	1.0000	0	1.0000	*1.0000*
	+-	17	21	42	15	27	15	0.0041	17	0.0286	*0.0082***
Med. firm:	+	35	22	35	15	7	7	0.0058	15	0.0033	*0.0067***
	-	1	1	7	7	13	1	1.0000	1	1.0000	*1.0000*
	+-	36	23	42	22	20	20	0.0010	22	0.0109	*0.0020***
Large firm:	+	10	23	13	5	9	5	0.0313	9	0.0231	*0.0463**
	-	2	1	22	25	6	1	1.0000	2	1.0000	*1.0000*
	+-	12	24	35	30	15	12	0.0126	15	0.0453	*0.0251**
Panel B: modified Pearson correlation test											
Small firm:	+	15	22	40	13	16	13	0.0003	15	0.0033	*0.0005***
	-	0	0	1	1	10	0	1.0000	0	1.0000	*1.0000*
	+-	15	22	41	14	26	14	0.0058	15	0.0453	*0.0116**
Med. firm:	+	34	21	33	14	7	7	0.0058	14	0.0044	*0.0087***
	-	1	1	7	9	15	1	1.0000	1	1.0000	*1.0000*
	+-	35	22	40	23	22	22	0.0006	22	0.0109	*0.0012***
Large firm:	+	11	24	12	6	9	6	0.0126	9	0.0232	*0.0251**
	-	2	1	23	25	6	1	1.0000	2	1.0000	*1.0000*
	+-	13	25	35	31	15	13	0.0084	15	0.0453	*0.0168**
Panel C: Kendall tau test											
Small firm:	+	16	20	41	9	16	9	0.0017	16	0.0026	*0.0033***
	-	0	0	1	0	14	0	1.0000	0	1.0000	*1.0000*
	+-	16	20	42	9	30	9	0.0529	16	0.0358	*0.0715*
Med. firm:	+	33	24	49	14	6	6	0.0126	14	0.0044	*0.0087***
	-	0	1	5	7	17	0	1.0000	1	1.0000	*1.0000*
	+-	33	25	54	21	23	21	0.0008	23	0.0092	*0.0015***
Large firm:	+	9	19	26	7	10	7	0.0058	9	0.0232	*0.0116**
	-	4	1	15	25	10	1	1.0000	4	0.3815	*0.7629*
	+-	13	20	41	32	20	13	0.0084	20	0.0156	*0.0168**

This Table reports the number of rejections of Null Hypotheses VI (VIA, VIB). If NT and BAB are the sole sources of stock return autocorrelation, the correlation between the return of the individual stock and the return of the SPDRs must be less than or equal to zero. Our Null Hypotheses VI (VIA, VIB) are that the correlation of each of the individual stock returns and the return of the SPDRs is zero (nonpositive, nonnegative). The daily return of each individual stock on day *d* is computed in the conventional way: the price at the final trade on day *d*, minus the price at the last trade prior to day *d*, divided by the price at the last trade prior to day *d*. We compute the correlation between the return of stock *i* on day *d*+1 (in other words, the return from the final trade of the stock on day *d* to the final trade of the stock on day *d*+1) with the return of the SPDRs over the interval from the time of the last trade of the SPDRs on day *d*-1 through the time of the last trade of the *stock* on day *d*. If a stock does not trade on day *d* or the stock does not trade on day *d*+1, we omit the data from our calculation. Our sample consists of 300 NYSE-listed sample firms, stratified into the three groups. We use three methods to test whether the correlation is zero: the Pearson correlation test (Panel A), a modified Pearson correlation test (Panel B), and the Kendall tau test (Panel C). Our modified Pearson test uses Andrew's (1991) heteroskedasticity and autoregression consistent (HAC) covariance estimator for the estimation and test of the correlation coefficient. We obtain the Andrew's HAC covariance using the quadratic spectral (QS) kernel with automatic bandwidth selection method. We use transaction data from the TAQ database over the sample period from January 4, 1993 to December 31, 2002. + and - denote the numbers of stocks with statistically significant positive and negative correlation with the SPDRs, at the 2.5% level; +- denotes the numbers of stocks with statistically significant correlation in a two-sided test, at the 5% level. X_1 denotes the first order statistic (minimum) of the observations for the five two time-horizon subperiods, while X_2 denotes the second order statistic (second smallest). p_1 and p_2 denote the probability that X_1 and X_2, respectively, would exceed the observed value in a nonparametric test using only the fact that the numbers in + and – are nonnegative random variables with expectation 2.5. $p_3 = 2 \min\{p_1, p_2\}$ is an upper bound on the probability that either X_1 or X_2 would exceed the observed value. ** and * denote significance at the 1% and 5% levels using p_3.

Table 10
ETFs—large firms only—lag at least three minutes
(Null Hypotheses VI, VIA, VIB)

Portfolio		Correlation of daily *individual* stock returns and SPDRs									
		93-94	95-96	97-98	99-00	01-02	X_1	p_1	X_2	p_2	p_3
Panel A: Pearson correlation test											
Large firm:	+	9	22	12	5	8	5	0.0313	8	0.0358	*0.0625*
	-	3	1	22	26	6	1	1.0000	2	1.0000	*1.0000*
	+-	12	23	34	31	14	12	0.0126	14	0.0581	*0.0251**
Panel B: modified Pearson correlation test											
Large firm:	+	9	22	13	6	9	6	0.0126	9	0.0232	*0.0251**
	-	3	1	23	25	6	1	1.0000	3	0.8038	*1.0000*
	+-	12	23	36	31	15	12	0.0126	15	0.0453	*0.0251**
Panel C: Kendall tau test											
Large firm:	+	8	18	26	7	10	7	0.0058	8	0.0358	*0.0116**
	-	5	1	15	25	10	1	1.0000	5	0.1875	*0.3750*
	+-	13	19	41	32	20	13	0.0084	19	0.0189	*0.0168**

This Table reports the number of rejections of Null Hypotheses VI (VIA, VIB), for large firms only, when the SPDR trade is required to be at least 3 minutes prior to the closing trade of the stock. If NT and BAB are the sole sources of stock return autocorrelation, the correlation between the return of the individual stock and the return of the SPDRs must be less than or equal to zero. Our Null Hypotheses VI (VIA, VIB) are that the correlation of each of the individual stock returns and the return of the SPDRs is zero (nonpositive, nonnegative). The daily return of each individual stock on day d is computed in the conventional way: the price at the final trade on day d, minus the price at the last trade prior to day d, divided by the price at the last trade prior to day d. We compute the correlation between the return of stock i on day $d+1$ (in other words, the return from the final trade of the stock on day d to the final trade of the stock on day $d+1$) with the return of the SPDRs over the interval from the time of the last trade of the SPDRs on day $d-1$ through three minutes before the time of the last trade of the *stock* on day d. If a stock does not trade on day d or the stock does not trade on day $d+1$, we omit the data from our calculation. Our sample consists of 300 NYSE-listed sample firms, stratified into the three groups. We use three methods to test whether the correlation is zero: the Pearson correlation test (Panel A), a modified Pearson correlation test (Panel B), and the Kendall tau test (Panel C). Our modified Pearson test uses Andrew's (1991) heteroskedasticity and autoregression consistent (HAC) covariance estimator for the estimation and test of the correlation coefficient. We obtain the Andrew's HAC covariance using the quadratic spectral (QS) kernel with automatic bandwidth selection method. We use transaction data from the TAQ database over the sample period from January 4, 1993 to December 31, 2002. + and - denote the numbers of stocks with statistically significant positive and negative correlation with the SPDRs, at the 2.5% level; +- denotes the numbers of stocks with statistically significant correlation in a two-sided test, at the 5% level. X_1 denotes the first order statistic (minimum) of the observations for the five two time-horizon subperiods, while X_2 denotes the second order statistic (second smallest). p_1 and p_2 denote the probability that X_1 and X_2, respectively, would exceed the observed value in a nonparametric test using only the fact that the numbers in + and – are nonnegative random variables with expectation 2.5. $p_3 = 2 \min\{p_1, p_2\}$ is an upper bound on the probability that either X_1 or X_2 would exceed the observed value. ** and * denote significance at the 1% and 5% levels using p_3.

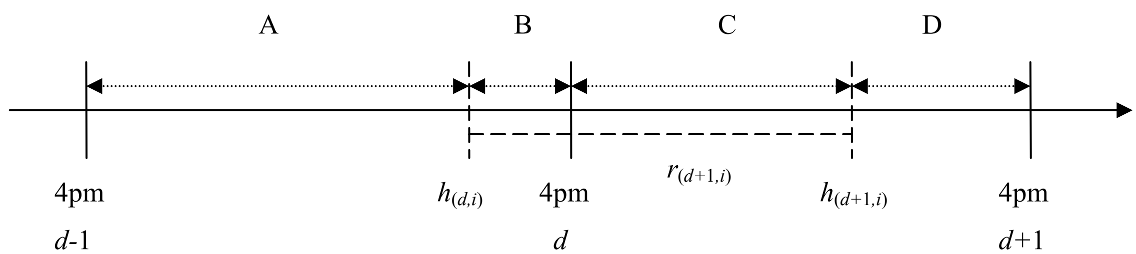

Fig. 1. Time diagram for Null Hypotheses VI, VIA, and VIB. Our Null Hypotheses VI (VIA, VIB) are that the correlation of each of the individual stock returns and the return of the SPDRs is zero (nonpositive, nonnegative). $r_{(d+1,i)}$ is the daily return of each individual stock on day d+1, computed in the conventional way: the price at the final trade on day d+1, minus the price at the last trade prior to day d+1, divided by the price at the last trade prior to day d+1. We compute the correlation between the return of stock i on day d+1 (in other words, the return from the final trade of the stock on day d to the final trade of the stock on day d+1, corresponding to the intervals B and C) with the return of the SPDRs over the interval from the time of the last trade of the SPDRs on day d-1 through the time of the last trade of the *stock* on day d, corresponding to the interval A. If a stock does not trade on day d or the stock does not trade on day d+1, we omit the data from our calculation.